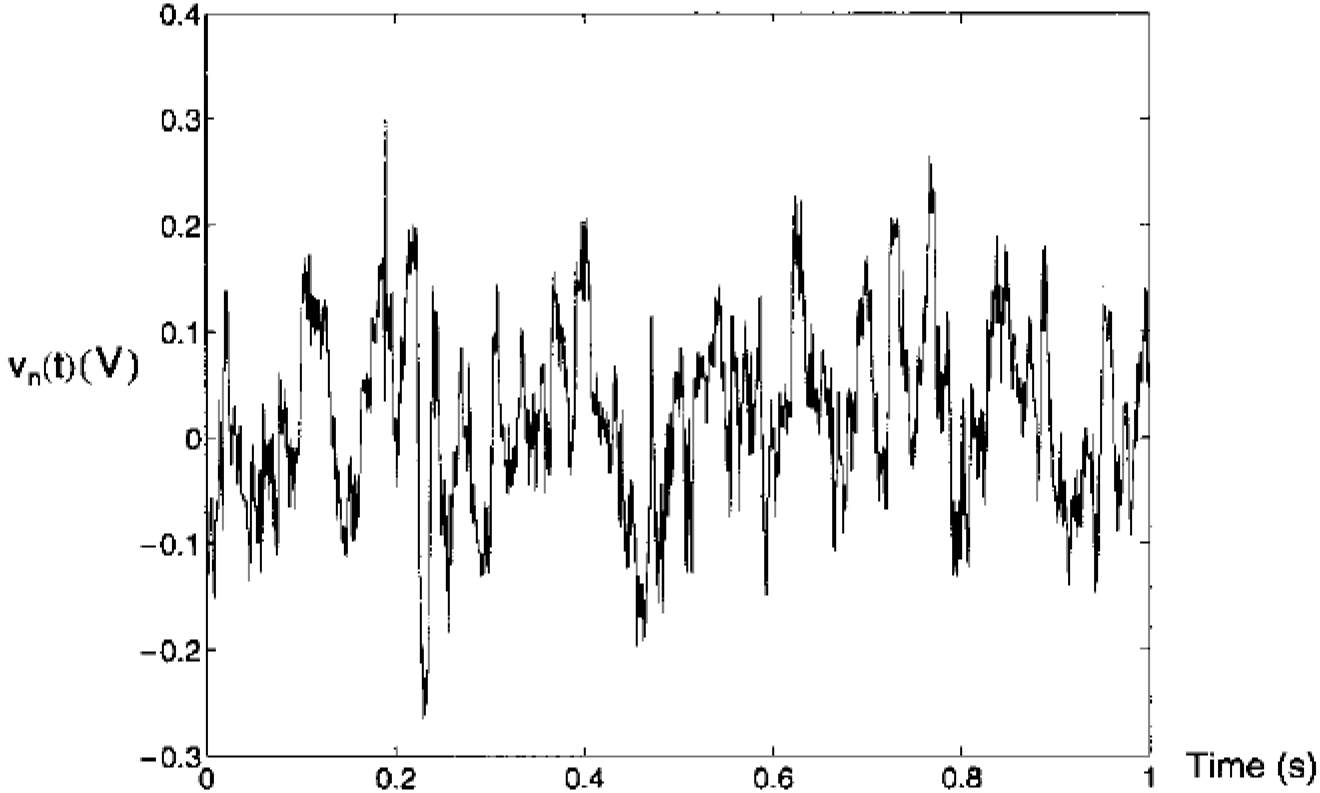
Noise Analysis and Modeling



Throughout this chapter we will assume all noise signals have a mean value of zero. We also assume that random signals are Ergodic.

Time-Domain Analysis

RMS (root mean square) Value is defined:

$$V_{n(rms)} \equiv \left[\frac{1}{T}\int_0^T v_n^2(t)\,dt\right]^{1/2}$$

where T is a suitable averaging time interval.

The normalized noise power is given by

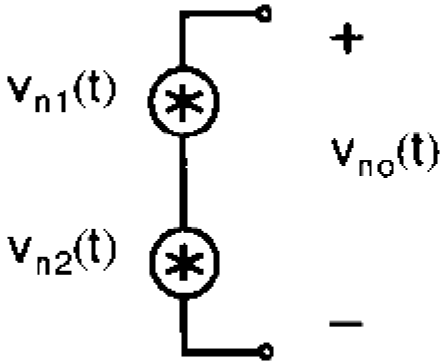
$$P_{diss} = \frac{V_{n(rms)}^2}{1\ \Omega} = V_{n(rms)}^2$$

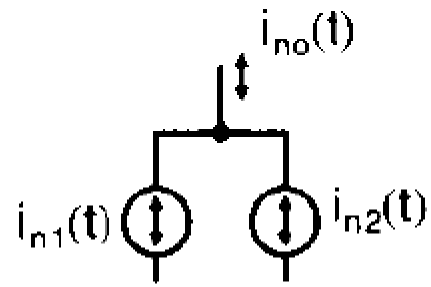
SNR is defined as follows:

$$SNR \equiv 10\log\left[\frac{\text{signal power}}{\text{noise power}}\right]$$

$$SNR = 10\log\left[\frac{V_{x(rms)}^2}{V_{n(rms)}^2}\right] = 20\log\left[\frac{V_{x(rms)}}{V_{n(rms)}}\right]$$

Noise Summation





$$v_{no}(t) = v_{n1}(t) + v_{n2}(t)$$

$$V^2_{no(rms)} = \frac{1}{T}\int_0^T [v_{n1}(t) + v_{n2}(t)]^2\, dt$$

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T v_{n1}(t)v_{n2}(t)\, dt$$

Definition of **correlation coefficient**:

$$C \equiv \frac{\frac{1}{T}\int_0^T v_{n1}(t)v_{n2}(t)\, dt}{V_{n1(rms)}V_{n2(rms)}}$$

Noise Summation

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + 2CV_{n1(rms)}V_{n2(rms)}$$

It can be shown that the correlation coefficient always satisfies the condition $-1 \le C \le 1$. Also, a value of $C = \pm 1$ implies the two signals are fully correlated, whereas $C = 0$ indicates the signals are uncorrelated.

In the case of two uncorrelated signals, the mean-squared value of their sum is given by

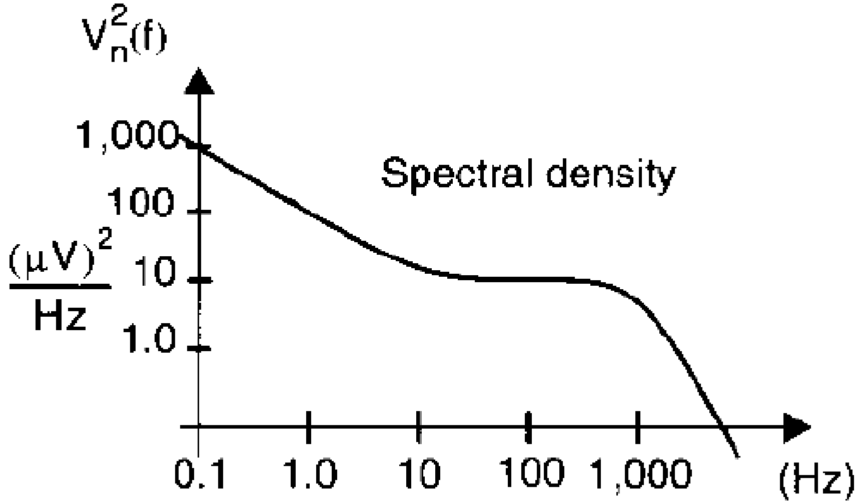
$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)}$$

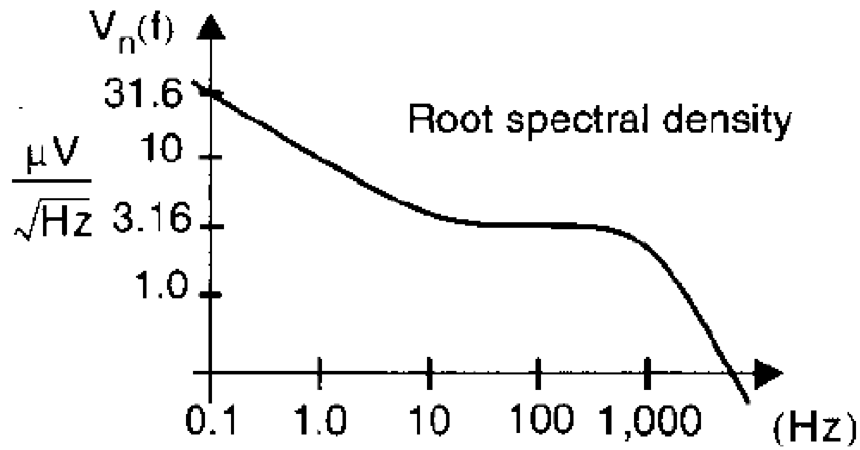
In the case of two fully correlated signals, the mean-squered value of their sum is

$$V^2_{no(rms)} = [V_{n1(rms)} \pm V_{n2(rms)}]^2$$

Frequency-Domain Analysis

Noise Spectral Density: (PSD)

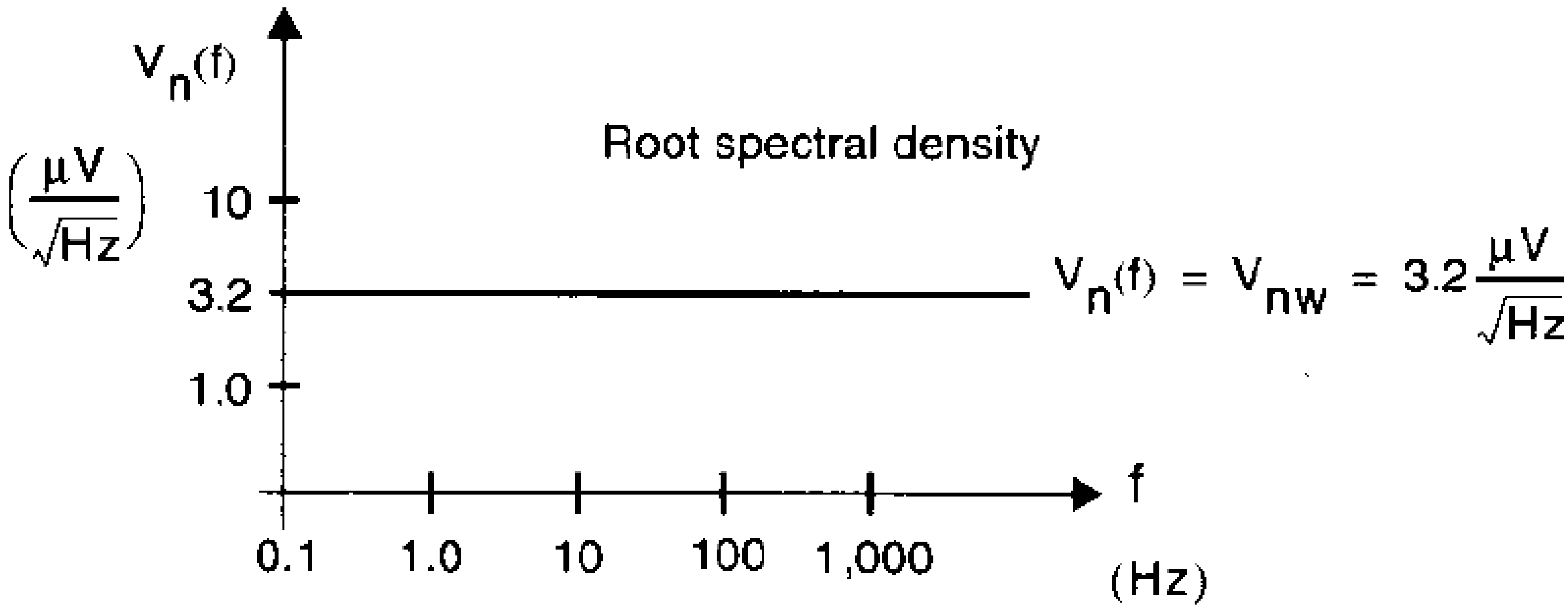




It is proven that:

$$V_{n(rms)}^2 = \int_0^{\infty} V_n^2(f)\, df$$

White Noise



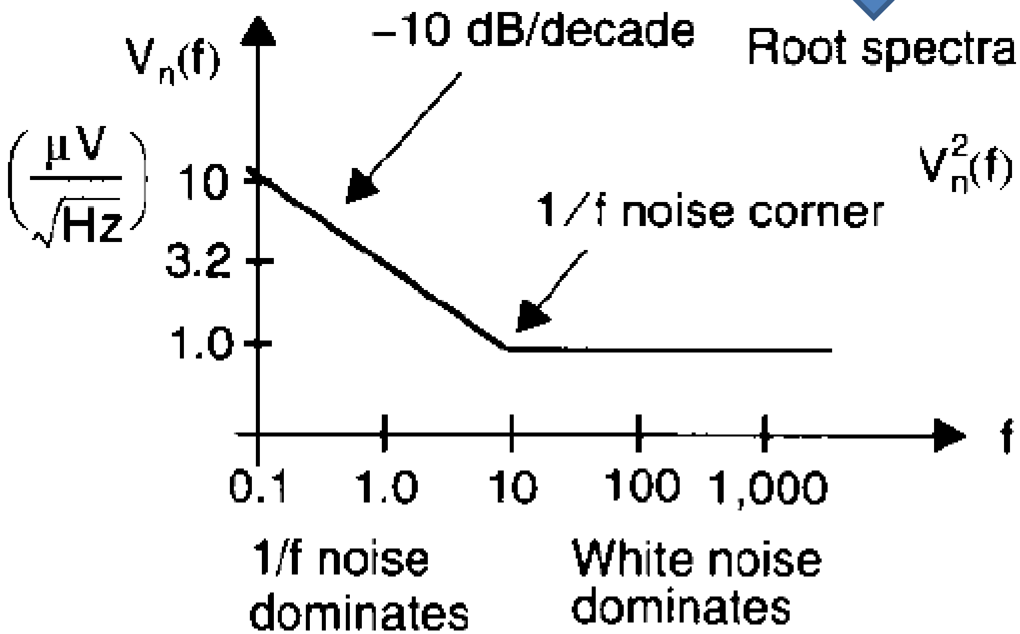
$$V_n(f) = V_{nw}$$

1/f or Flicker Noise

$$V_n^2(f) = \frac{k_v^2}{f}$$

$$V_n(f) = \frac{k_v}{\sqrt{f}}$$

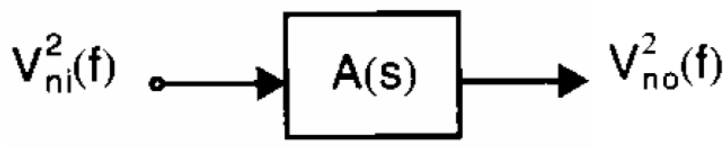
A noise signal that has both 1/f and white noise.



Root spectral density

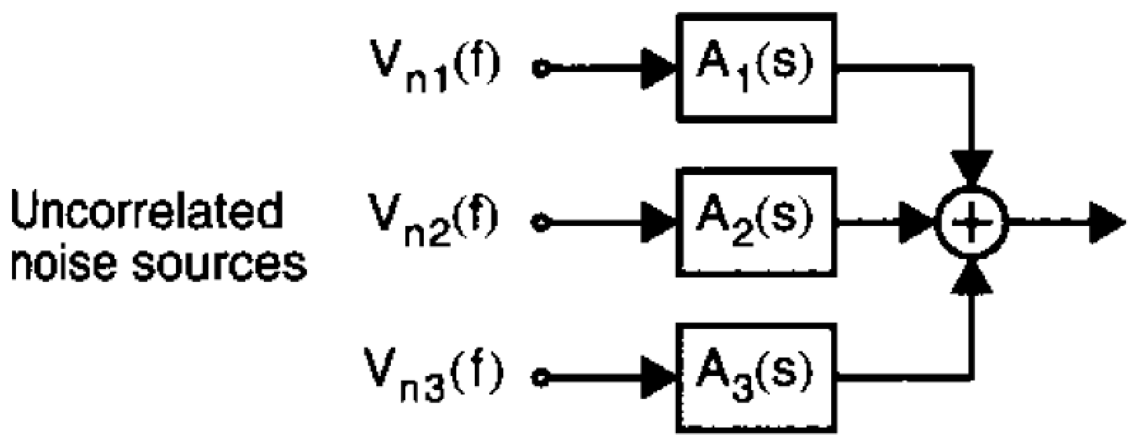
$$V_n^2(f) \approx \frac{(3.2 \times 10^{-6})^2}{f} + (1 \times 10^{-6})^2$$

Filtered Noise



$$V_{no}^2(f) = |A(j2\pi f)|^2 V_{ni}^2(f)$$

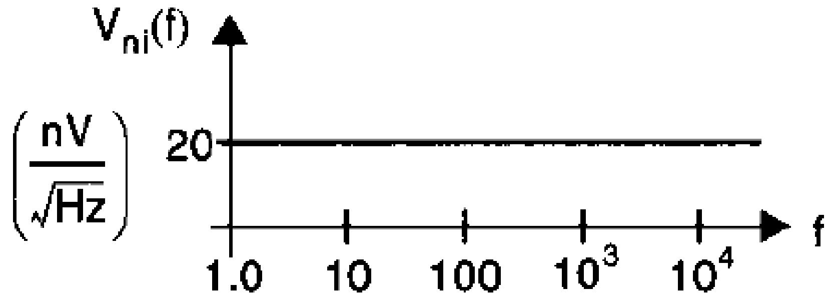
$$V_{no(rms)}^2 = \int_0^\infty |A(j2\pi f)|^2 V_{ni}^2(f)\, df$$

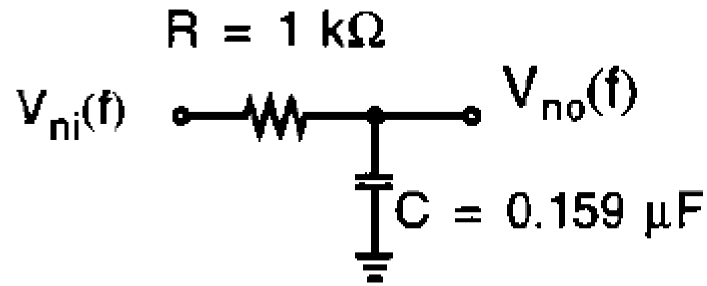


$$V_{no}^2(f) = \sum_{i=1,2,3} |A_i(j2\pi f)|^2 V_{ni}^2(f)$$

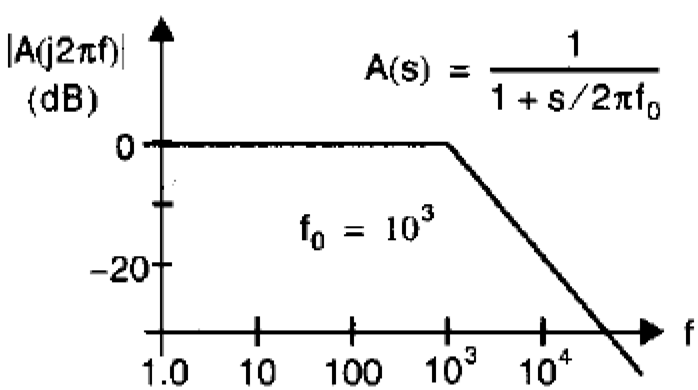
Example

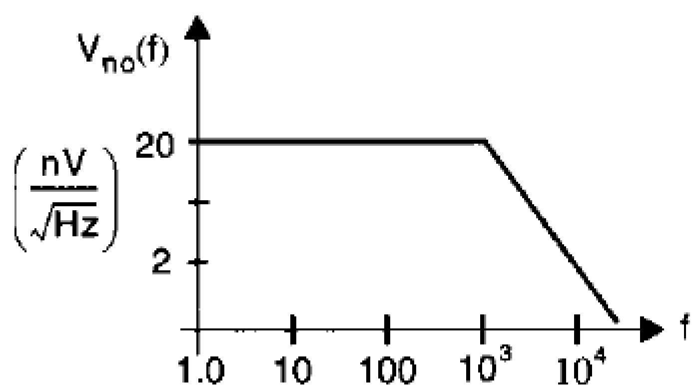
Assume a white noise at the input of the filter as shown bellow. What is the PSD of the output noise? What is the rms of the output noise?





$$f_0 = \frac{1}{2\pi \times 1K \times 0.159\mu} = 1kHz$$





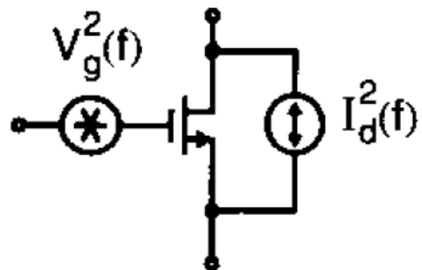
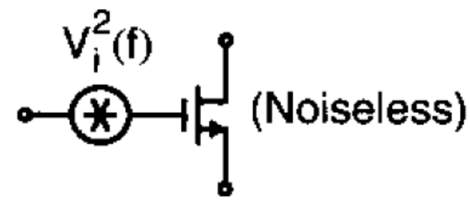
continued

$$V_{no}(f) = \frac{20 \times 10^{-9}}{\sqrt{1 + \left(\frac{f}{f_0}\right)^2}}$$

$$V_{no(rms)}^2 = \int_0^{+\infty} \frac{400 \times 10^{-18}}{1 + \left(\frac{f}{f_0}\right)^2} = 400 \times 10^{-18} f_0 \arctan \frac{f}{f_0} \Big|_0^{+\infty} = 400 \times 10^{-18} \times 1000 \times \frac{\pi}{2} = 6.28 \times 10^{-13}$$

$$V_{no(rms)} = 0.8 \mu V$$

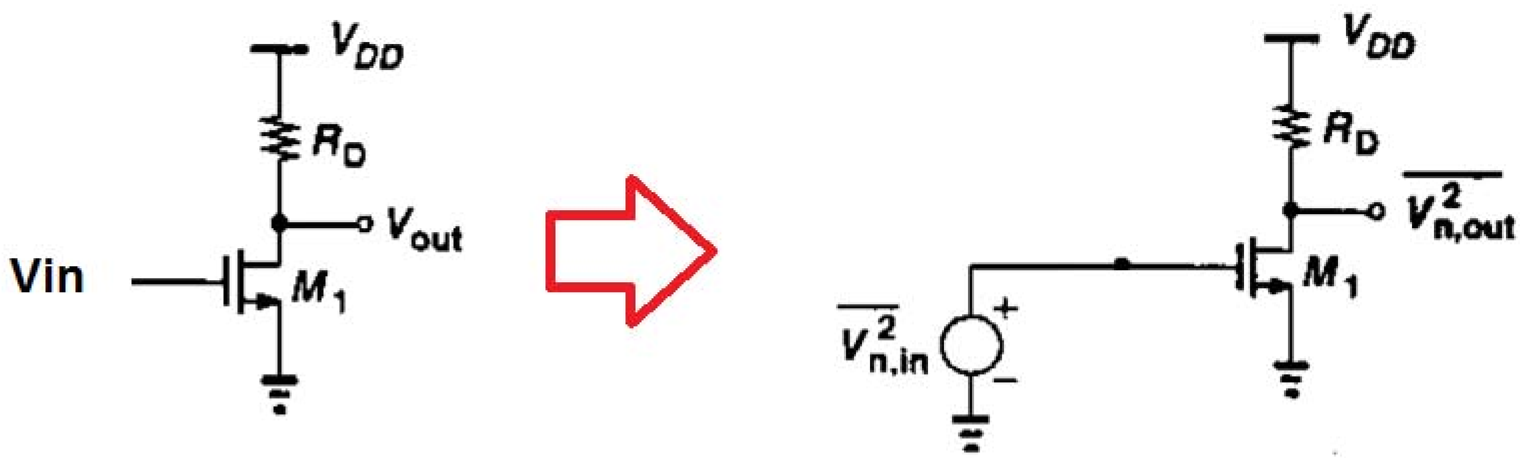
Noise Models for Circuit Elements (MOSFETS)

MOSFET  (Active region)	 $V_g^2(f) = \frac{K}{WLC_{ox}f}$ $I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m$	 $V_i^2(f) = 4kT\left(\frac{2}{3}\right)\frac{1}{g_m} + \frac{K}{WLC_{ox}f}$ Simplified model for low and moderate frequencies

$$K = 10^{-25} V^2 F$$

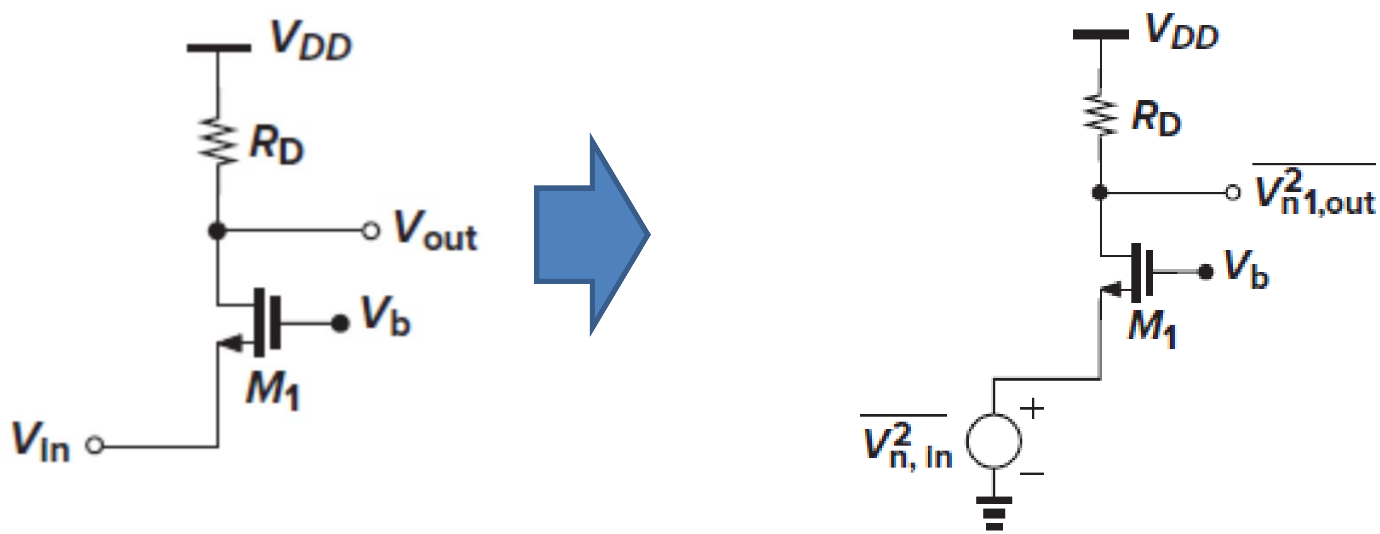
$$I_d^2(f) = 4kT\gamma g_m$$

Noise Calculation of C.S Amplifier



Neglecting the flicker noise, we have: $\overline{V_{n,in}^2} = 4kT\left(\frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D}\right)$

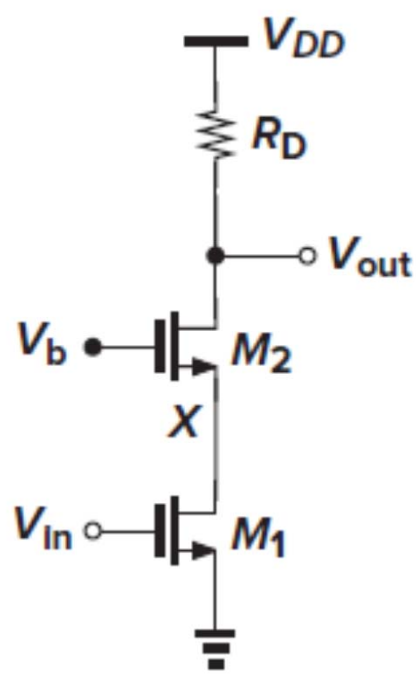
Noise Calculation of C.G Amplifier



Neglecting the flicker noise and considering the body effect, we have:

$$\overline{V_{n,in}^2} = \frac{4kT(\gamma g_m + 1/R_D)}{(g_m + g_{mb})^2}$$

Noise Calculation of Cascode Amplifier

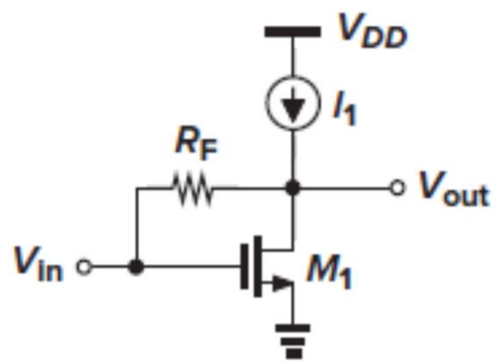


It is proven that we can neglect the noise of M2. So we have:

$$\overline{V_{n,in}^2}|_{M1,RD} = 4kT\left(\frac{\gamma}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right)$$

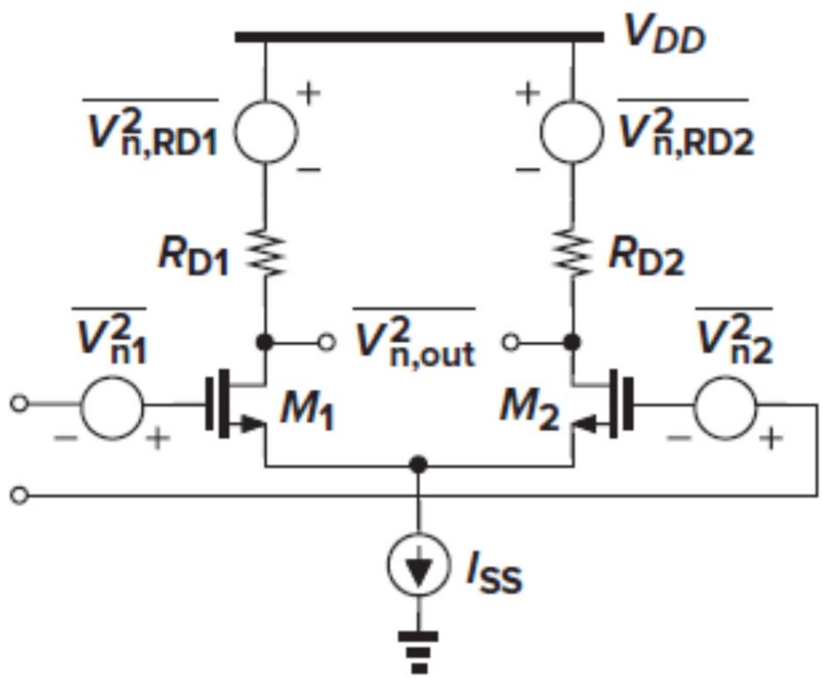
Example

Determine the input-referred noise voltage of the amplifier. Assume that I_1 is noiseless and $\lambda = 0$.



$$\overline{V_{n,in}^2} = \frac{\frac{4kT}{R_F} + 4kT\gamma g_m}{(g_m - \frac{1}{R_F})^2}$$

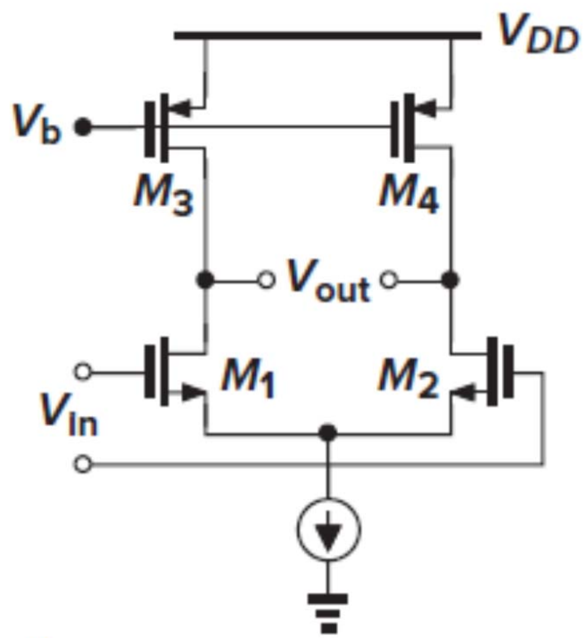
Noise Calculation of Fully Differential Amplifier



$$\overline{V_{n,in,tot}^2} = 8kT\left(\frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D}\right)$$

Example

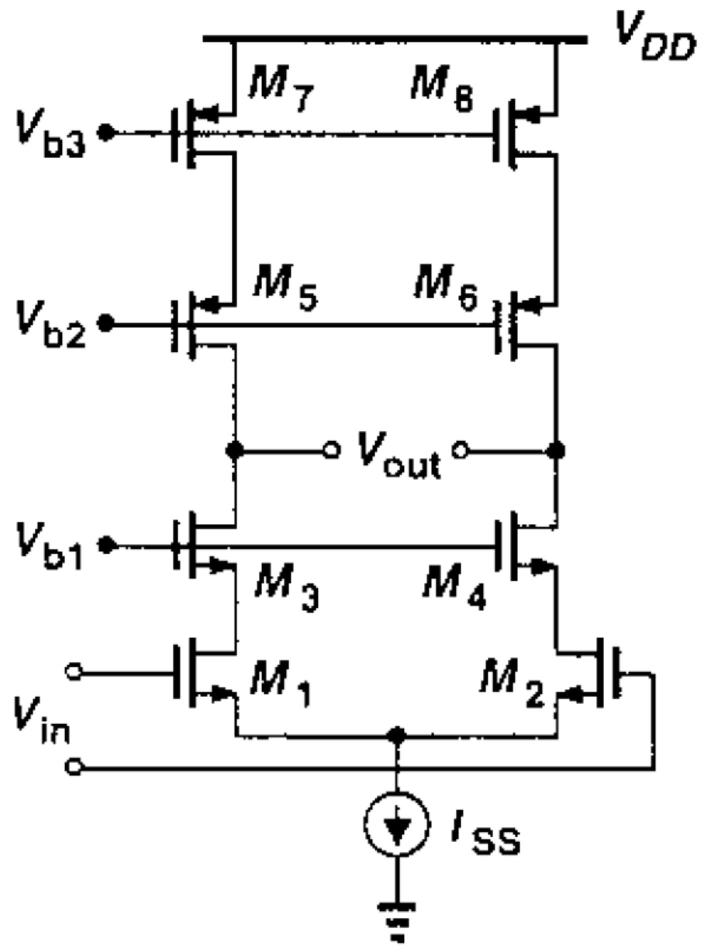
Calculate the input referred noise.



$$\overline{V_{n,in}^2} = 2\overline{V_{n1}^2} + 2\frac{g_{m3}^2}{g_{m1}^2}\overline{V_{n3}^2}$$

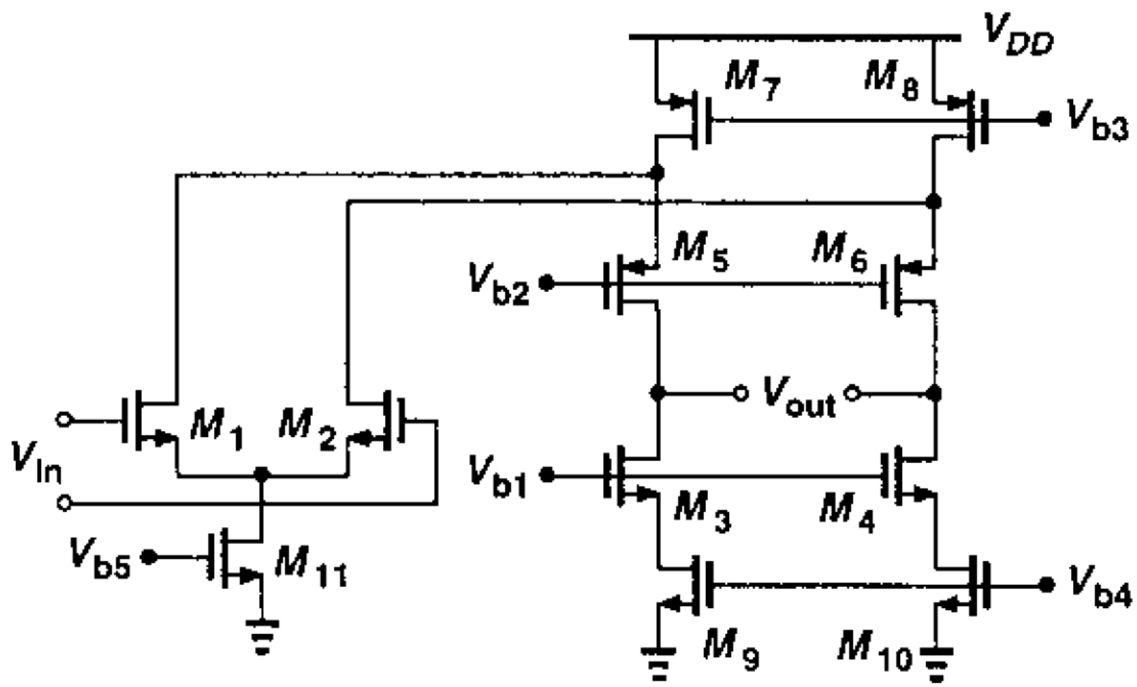
$$\overline{V_{n,in}^2} = 8kT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m3}}{g_{m1}^2}\right)$$

Telescopic Opamp



$$\overline{V_n^2} = 4kT\left(2\frac{\gamma}{g_{m1,2}} + 2\frac{\gamma g_{m7,8}}{g_{m1,2}^2}\right)$$

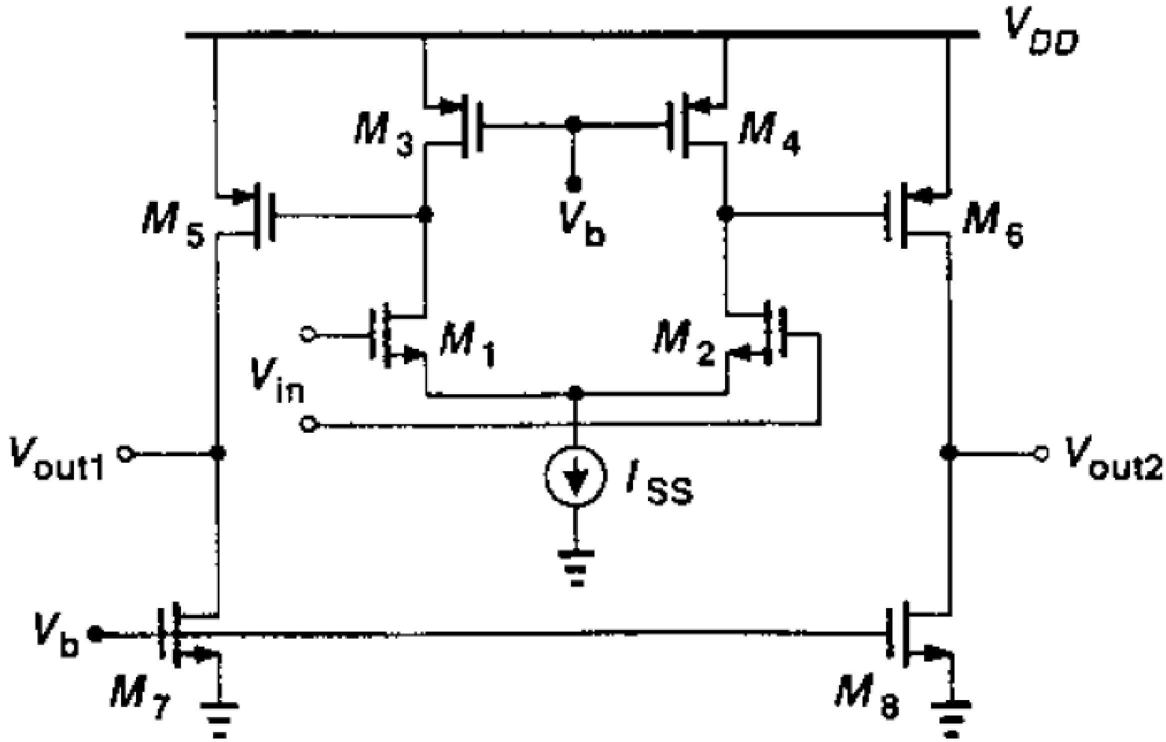
Folded-Cascode Opamp



Neglecting the flicker noise, we have:

$$\overline{V_{n,tot}^2} = 8kT\gamma \frac{1}{g_{m1}^2}\left[g_{m1} + g_{m3} + \frac{g_{m5} + g_{m7}}{g_{m5}^2 (r_{O1} \| r_{O3})^2}\right]$$

Two-Stage Opamp



Neglecting the flicker noise, we have:

$$\overline{V_{n,tot}^2} = \frac{16kT}{3}\frac{1}{g_{m1}^2}\left[g_{m1} + g_{m3} + \frac{g_{m5} + g_{m7}}{g_{m5}^2(r_{O1}\|r_{O3})^2}\right].$$