Chapter 12
A Mathematical Analysis Around Capacitive Characteristics of the Current of CSCT: Optimum Utilization of Capacitors of Harmonic Filters

Mohammad Golkhah and Mohammad Tavakoli Bina

Abstract A new shunt reactive power compensator, CSCT, is presented and introduced in this paper. Mathematical analysis of harmonic content of the current of CSCT is performed and use of a winding with additional circuit has been presented as a solution to suppress these harmonics.

Keywords CSCT · Harmonic filter · Reactive power compensation · Thyristor controlled transformer

12.1 Introduction

CSCT stands for "Controlled Shunt Compensator of Transformer type" [1–4]. A general scheme of this compensator is presented in Fig. 12.1. This configuration is a transformer with three windings. NW is network winding which is connected to the network and is the main winding of the compensator. CW is the second winding to which a thyristor valve and a parallel voltage circuit breaker are connected and is called CW briefly. The third winding is compensating winding which is indicated by ComW in Fig. 12.1. Two highest harmonic filters and a capacitor bank are connected to this winding. It is important to note that CSCT is a three phase compensator. The connection of NWs of three phases is star and the neutral is grounded. Control windings' connection is as same as network windings of three phases. However compensating windings can be delta in connection together.

When the thyristor is opened all the magnetic flux passes through the magnetic core leading to a minimum reluctance, maximum inductance and a capacitive current in NW according to the value of capacitor bank and eventually generate reactive power to the network. On contrary, when the thyristor is closed the flux is subjected to pass through air gap including all of the windings. Hence the reluctance, inductance and the current of NW will be maximal, minimal and maximal inductive (the rated value) respectively.

M. Golkhah (✉) and M.T. Bina
Electrical Engineering Department, K. N. Toosi University of Technology, Tehran, Iran
e-mail: goulkhah_elect@yahoo.co.uk

S.-I. Ao et al. (eds.), *Advances in Machine Learning and Data Analysis*, Lecture Notes in Electrical Engineering 48, DOI 10.1007/978-90-481-3177-8_12,

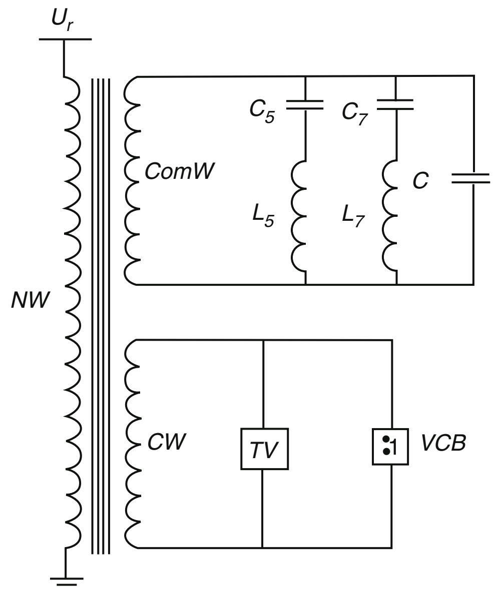

Fig. 12.1 General scheme of a CSCT: TV, thyristors valve; VCB, vacuum circuit breaker; C5-L5 and C7-L7 filters of fifth and seventh harmonics; C, additional capacitor bank

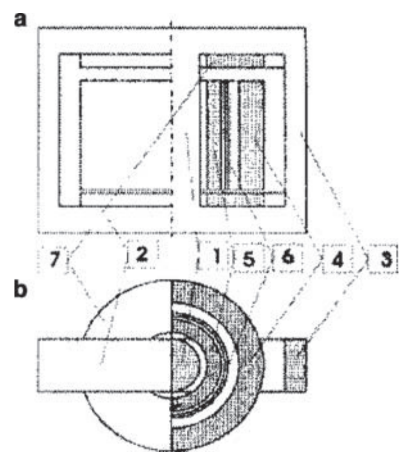

Fig. 12.2 Mono phase diagram of CSCT: **a**, view form side; **b**, view form above: 1, a main core; 2, yokes; 3, lateral yokes; 4, NW; 5, CW; 6, ComW; 7, magnetic shunts

Since two modes lead to two different signs in the reactive power, we can claim that the compensator is bilateral. However the capacitor can be removed from the compensating winding still having bilateral reactive power compensation if the relationships of designing elements of harmonic filters be correctly performed. These relations are calculated and presented below.

Figure 12.2 represents one phase of the transformer with mentioned three windings. The winding close to the main core is CW, the outer winding is NW and interlayed winding is ComW.

12.2 Mathematical Analysis

Flowing capacitive current in the network winding of CSCT under open circuit condition of CW, which defines the capacitive conductance of harmonic filters of fifth and seventh in the main frequency, has prompted the possibility of bilateral action of the compensator with increased capacitive component of filters of the higher harmonics (see Fig. 12.3).

Impedance of the compensator when thyristors are completely closed is defined by (12.1).

$$\begin{aligned} X_{Leq} &= X_1 + \frac{X_2 X_3}{X_2 + X_3} = \delta X_{12} + \frac{(1-\delta)\, X_{12} X_3}{(1-\delta)\, X_{12} + X_3} \\ &= X_{12}\left[\delta + \frac{(1-\delta)}{1 + \frac{X_{12}}{X_3}(1-\delta)}\right] > X_{12} \end{aligned} \tag{12.1}$$

This value is greater than the corresponding impedance of controlled shunt *reactor* of transformer type, CSRT.

$$\begin{aligned} X_{L.eq.CSRT} &= X_1 + \frac{X_2 X_{f.eq}}{X_2 + X_{f.eq}} = \delta X_{12} + \frac{(1-\delta)\, X_{12} X_{f.eq}}{(1-\delta)\, X_{12} + X_{f.eq}} \\ &= X_{12}\left[\delta + \frac{(1-\delta)}{1 + \frac{X_{12}}{X_{f.eq}}(1-\delta)}\right] \approx X_{1\,2} \end{aligned} \tag{12.2}$$

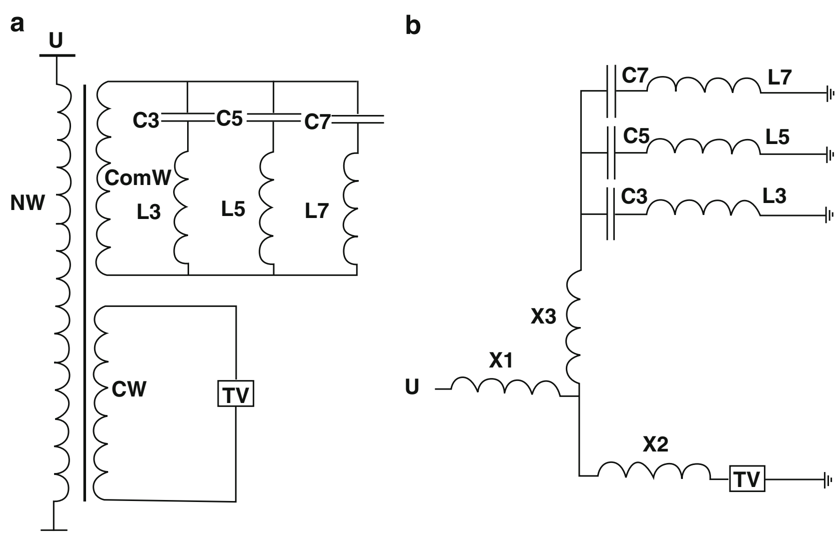

Fig. 12.3 Schematic single-line diagram (**a**) and equivalent circuit (**b**) of CSCT without any capacitor bank

To provide the rated current by CSCT as same as CSRT, the impedances X_{12}, X_{13} and X_{23} should be lowered. For instance this objective can be achieved by reducing the number of turns in windings or increasing the height of magnetic window.

In a general view, the solution to this problem can be carried out on the basis of two equations made for locked and opened thyristors. In open thyristors condition, we want the capacitive current flowing through the network winding of CSCT to be equal to its rated current, then

$$I_c = \frac{U_{ph}}{\delta X_{12} + X_c} = -\alpha \frac{U_{ph}}{X_{L.nom}} \tag{12.3}$$

Where X_{Lnom} is the essential rated impedance of compensator in inductive mode of operation and α is required correlation between rated capacitive and inductive currents of the compensator.

The essential value of X_C is obtained from (12.4).

$$X_c = \frac{1}{\alpha}\left(X_{L.nom} + \alpha\delta \cdot X_{12}\right) \tag{12.4}$$

Let's now suppose that the inductive current through network winding of the compensator is equal to its rated current with completely closed thyristors:

$$I_L = \frac{U_{ph}}{\delta X_{12} + \frac{(1-\delta)X_{12}X_c}{(1-\delta)X_{12}+X_c}} = \frac{U_{ph}}{X_{L.nom}} \tag{12.5}$$

Then we have from (12.5):

$$X_{L.nom} = \delta X_{12} + \frac{(1-\delta)\,X_{12}X_c}{(1-\delta)\,X_{12} + X_c} \tag{12.6}$$

After transposing and substituting Xc from (12.4), we have:

$$\alpha\delta^2 X_{12}^2 + X_{12}X_{Lnom}\left(1 + \alpha - 2\alpha\delta\right) - X_{L.nom}^2 = 0 \tag{12.7}$$

The solution of (12.7) specifically when the rated currents in inductive and capacitive modes of operation are equalized ($\alpha = 1$) obtains:

$$X_{12} = X_{L.nom}\frac{\sqrt{1-2\delta+2\delta^2}+\delta-1}{\delta^2} = X_{L.nom}\frac{\sqrt{1-2\delta\cdot(\delta-1)}+\delta-1}{\delta^2} \tag{12.8}$$

$$\begin{aligned} X_C &= -X_{L.nom}\frac{\delta+\sqrt{1-2\delta\cdot(\delta-1)}+\delta-1}{\delta} \\ &= -X_{L.nom}\frac{2\delta-1+\sqrt{1-2\delta\cdot(\delta-1)}}{\delta} \end{aligned} \tag{12.9}$$

For example when $\delta = 0.6$

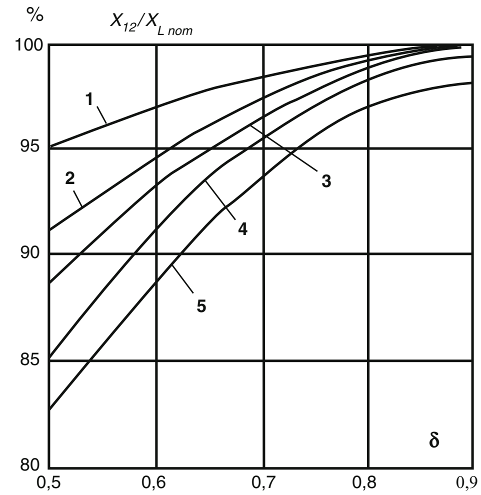

Fig. 12.4 Dependences of relative value of reactance between NW and CW windings on relative value of reactance between NW and ComW $\delta = X_{13}/X_{12}$ at different ratios of rated currents in distribution and consumption conditions of reactance: $1 - \alpha = 0.2$; 2–0.4; 3–0.6; 4–0.8; 5–1.0

$$X_{12} = X_{L.nom}\frac{\sqrt{0.52} - 0.4}{0.36} = 0.891X_{L.nom} \text{ and}$$
$$X_c = -1.535X_{L.nom} = -1.72X_{12}$$

Since δ varies between 0.5–0.8, the rated inductance $X_{L.nom}$ is greater than X_{12} under any value of α. Hence the compensator's impedance under closed CW condition without filters is less than 100% unlike from CSRT (see Fig. 12.4).

It means that under the same rated inductive current and equal dimensions, which define X_{12}, the number of turns in windings of the compensator is less than that of CSRT. The ratio $X_{12}/X_{L.nom}$ is increased by increasing δ, and decreased when α is increased. Hence, higher is the value of α, smaller is the number of turns in the windings of CSCT in comparison with CSRT with the same capacity.

Reduction in the number of coils leads to increment in magnetic flux when thyristors are locked and when completely opened. Hence, in CSCT the active cross-section of magnetic conductor steel is greater than that of CSRT with the same capacity.

Further, we shall define the essential capacitance of highest harmonics filters to provide the relation α from the obtained value of Xc and known ratios for filters. The equivalent impedance of filters is defined by the ratio:

$$X_1 + X_{f.eq} = -\frac{X_{L.nom}}{\alpha}$$

So we have:

$$X_{f.eq} = \frac{X_{L.nom}}{\alpha}\left[1 + \frac{\sqrt{(1+\alpha-2\alpha\delta)^2 + 4\alpha\delta} - (1+\alpha-2\alpha\delta)}{2\delta}\right] \qquad (12.10)$$

Equivalent impedance of filters on power frequency is defined by the impedance of all installed filters and it will be defined by the ratio:

$$\frac{1}{X_{f.eq}} = \sum_{k=3}^{n} \frac{1}{X_{f.eq}} = \sum_{k=3}^{n} \frac{\omega \cdot C_k k^2}{1-k^2} \tag{12.11}$$

When using the third, fifth and seventh harmonic filters (if for some reason the compensation winding of the three phases is not in delta connection), we obtain:

$$\frac{1}{X_{f.eq}} = \omega\left(C_3\frac{9}{8} + C_5\frac{25}{24} + C_7\frac{49}{48}\right) = \omega C_3\left(1125 + 1.04\frac{C_5}{C_7}\right) = 1.5\omega C_3 \tag{12.12}$$

Whence

$$C_3 = \frac{1}{1.5\omega \cdot X_{f.eq}} = \frac{\alpha}{1.5\omega X_{L.nom}\left(1 + \frac{\alpha\delta \cdot X_{12}}{X_{L.nom}}\right)} \tag{12.13}$$

and C5 = 0.26C3; C7 = 0.105C3.

Voltage of capacitors of filters equals:

$$\Delta U_{c.f} = 1.1U_{ph}\left[1 + \frac{\sqrt{(1+\alpha-2\alpha\delta)^2 + 4\alpha\delta^2} - (1+\alpha-2\alpha\delta)}{2\delta}\right] \tag{12.14}$$

and the ratio of capacitor power of filters to the rated capacitive power of capacitors is:

$$\frac{Q_{c.f}}{Q_{C.CSCT}} = 1.1\left[1 + \frac{\sqrt{(1+\alpha-2\alpha\delta)^2 + 4\alpha\delta^2} - (1+\alpha-2\alpha\delta)}{2\delta}\right] \tag{12.15}$$

Corresponding dependences of ratios $Q_{c.f}/Q_{c.csct}$ are shown in Fig. 12.5.

Exceeding the voltage of compensating winding in comparison with the voltage defined by the transformation ratio leads to increment of the magnetic flux in the core. In order to eliminate the saturation of the core, it is essential to increase the active cross-section of the core proportional to the voltage increase (Fig. 12.6).

When the thyristors are opened, the current in branch 2 (Fig. 12.3) equals to zero and the voltage in the thyristors equals to the voltage in ComW (certainly in view of the ratio of the number of turns in CW and ComW ($K_{T.2-3} = K_{T.1-3}/K_{T.1-2}$). Consequently, with equal number of turns in ComW and CW, rated voltage of CW is defined by formula 12.14. With difference in number of turns in ComW and CW, the rated voltage in CW

$$U_{2nom} = \Delta U_{C.f}\, K_{T.2-3}$$

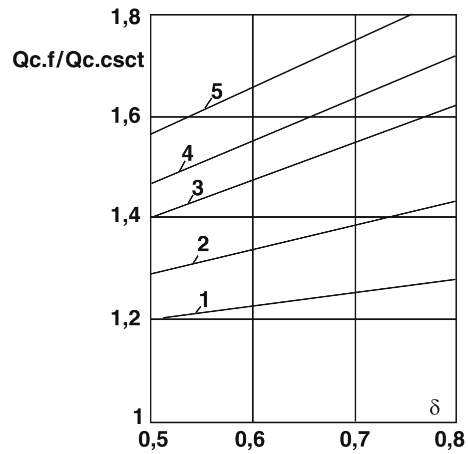

Fig. 12.5 Dependences of amount of third, fifth and seventh harmonics capacitor power – rated capacitive power of CSCT ratio on the relative values of inductive resistance of CSCT between ComW and CW $\delta = X_{13}/X_{12}$ at different values of $\alpha = Q_{C.f}/Q_{L.CSCT}$. ComW of the three phases are in Y-connection with neutral terminal

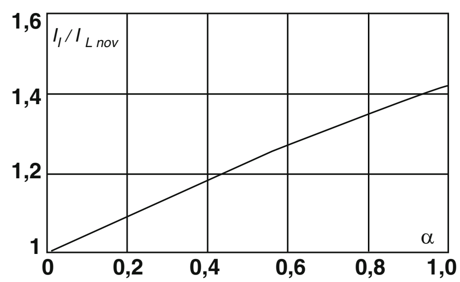

Fig. 12.6 Dependence of maximum current of CW on α when $\delta = 0.5$

The rated voltage of CW is chosen based on the appropriate (by technical and economic considerations) current through the thyristors. We shall obtain the relationship between rated current of CW and voltage in NW (see Fig. 12.3):

$$I_{2.nom}X_2 = U_{ph} - I_{L.nom}X_1$$

or

$$I_{2nom} = \frac{\delta}{1-\delta} I_{Lnom} \cdot \frac{1 + \alpha - \sqrt{(1 + \alpha - 2\alpha\delta)^2 + 4\alpha\delta^2}}{\sqrt{(1 + \alpha - 2\alpha\delta)^2 + 4\alpha\delta^2} - (1 + \alpha - 2\alpha\delta)} \tag{12.16}$$

For example when $\delta = 0.5$ according to (12.16)

$$\frac{I_{2nom}}{I_{Lnom}} = \sqrt{1 + \alpha}$$

The greater the ratio between rated capacitive and inductive currents α, the more the maximal current in CW increases relative to the rated inductive current in NW (subject to transformation coefficient).

Equating rated current of CW to the chosen rated current of thyristors, IT.nom, we get the necessary transformation coefficient:

$$K_{T.1-2} = \frac{I_{T.nom}}{I_{L.nom}} \cdot \frac{1-\delta}{\delta} \cdot \frac{1+\alpha-\sqrt{(1+\alpha-2\alpha\delta)^2+4\alpha\delta^2}}{\sqrt{(1+\alpha-2\alpha\delta)^2+4\alpha\delta^2}-(1+\alpha-2\alpha\delta)} \quad (12.17)$$

It can be found from (12.17) that the transformation coefficient of CSCT is smaller than that of CSRT with the same capacity and it depends on α and δ.

Thus increase in capacitance of capacitors enables to essentially expand the capacitor current regulation range of the compensator towards inductive current up to ±100% of rated CSRT current. Hence one thyristors block of CSRT calculated on the maximum current in CW is used. Quick closing of CW in nominal operating mode provides a negligible decrease in the rated current of the CSCT in comparison with the rated current of CSRT without supplementary capacitance. Moreover, CSCT scheme provides high efficient use of capacitance of filters by transforming reactors into static thyristors compensators. It is essential to note that all the regulations and additional instruments are carried out in the low voltage side, CW, which provides a relatively small additional costa in relation to CSRT. Since rated capacitive current flows through the network winding under locked thyristors, it is essential to provide closing conditions of thyristors in some part of the half-period of commercial frequency in order to reduce it. At a particular combustion angle of thyristors, the inductive current matches with the capacitive current and the network winding current reaches to zero. At further increase in combustion angle of thyristors, current becomes inductive and increases right up to the rated value when the combustion angle is 180° (complete conducting of thyristors). At zero current through the network winding, equivalent impedances of limbs 2 and 3 of CSCT three-beam schemes are the same. Hence equivalent impedance of thyristors limbs is equal to:

$$X_{2eq} = \frac{X_{Lnom}}{2\delta\alpha}\left[2\delta + \sqrt{(1+\alpha-2\alpha\delta)^2+4\alpha\delta^2}-(1+\alpha-2\alpha\delta)\right] \quad (12.18)$$

Since at zero current in the network winding of the compensator, voltage loss in inductive impedance $X_1 = \delta X_{12}$ equals to all voltages applied to any other two limbs of Fig. 12.3. Thus current in the thyristors limb equals to:

$$I_{2.0} = \frac{U_{ph}}{X_{2.eq}} = \frac{2\alpha\delta U_{ph}}{X_{L.nom}} \frac{1}{2\delta + \sqrt{(1+\alpha-2\alpha\delta)^2+4\alpha\delta^2}-(1+\alpha-2\alpha\delta)} \quad (12.19)$$

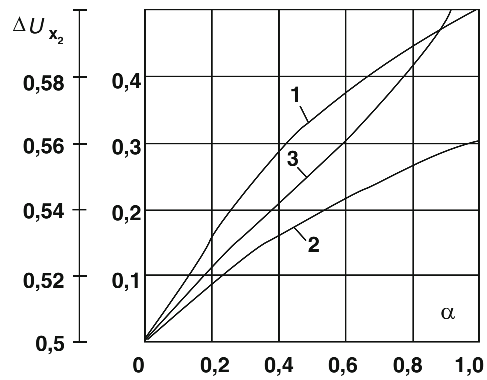

Fig. 12.7 Dependence on α of relative value of current through thyristor block when current in the network winding passes through zero (*curves 1,2*) at rated phase voltage (*curve 1*) and at voltage in CW under completely closed thyristors (*curve 2*), as well as the relative value of that voltage (*curve 3*)

And accordingly the relative value of current in thyristor limb under zero current in NW considering (12.16) will be:

$$\frac{I_{2.0}}{I_{2.nom}} = 2\alpha\,(1-\delta)\,\frac{2\delta + \sqrt{(1+\alpha-2\alpha\delta)^2 + 4\alpha\delta^2} - (1+\alpha-2\alpha\delta)}{1+\alpha-\sqrt{(1+\alpha-2\alpha\delta)^2+4\alpha\delta^2}} \times$$
$$\times \frac{1}{2\delta + \sqrt{(1+\alpha-2\alpha\delta)^2+4\alpha\delta^2} - (1+\alpha-2\alpha\delta)} \tag{12.20}$$

As seen, when the current passes through zero, the relative value of current in thyristor limb is defined by only two variables: α and δ. However calculations show that the ratio I2/I2nom does not depend on δ when I1 = 0 and its dependence on α is shown in Fig. 12.7, which is an approximated function (with inaccuracy not more the 2%).

$$I_2/I_{2nom} = 0.91\alpha \cdot e^{-0.6\alpha} \tag{12.21}$$

The ignition angle and correspondingly the combustion angle of thyristors when CSCT current passes through zero by the corresponding thyristor characteristic (Fig. 12.8) can define the relative value of current. However it follows to bear in mind that characteristic Fig. 12.8 was obtained at constant voltage and constant inductive impedance connected in series with thyristors. In the case of CSCT, inductive impedance $(1-\delta)X_{12}$ remains constant when any current passes through the thyristors and voltage at increased current increases right up to phase voltage at equal reactance of inductive and capacitive limbs.

In nominal inductive operating mode of CSCT, the current through inductance $X_2 = (1-\delta)X_{12}$ is defined by formula (12.16), from where voltage drop in that inductive resistance is:

$$\Delta U_{X_2} = L_{2nom} X_{12}\,(1-\delta) = \frac{U_{ph}}{2\alpha\delta}\left[1+\alpha-\sqrt{(1+\alpha-2\alpha\delta)^2+4\alpha\delta^2}\right] \tag{12.22}$$

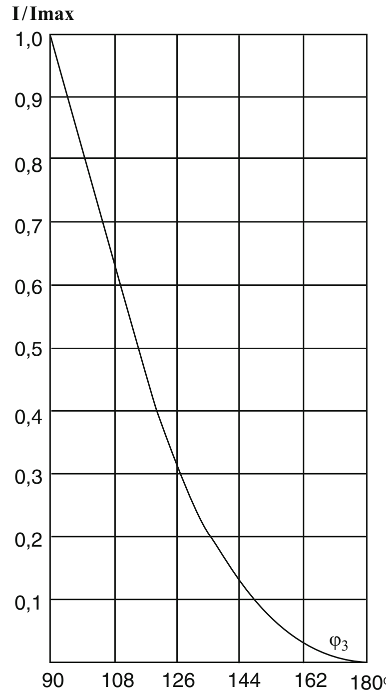

Fig. 12.8 Dependence of relative value of current through thyristor block on ignition angle of thyristors

and consequently the ratio of the voltage drop in the thyristor to the phase voltage changes with change in α (see Fig. 12.7), although this change is negligible (in the range of 18%).

In the Fig. 12.7, the dependence of $f(\alpha)$ brought to rated voltage in CW (when thyristors are completely opened) by multiplying the given curve 1 and the corresponding ratio of the voltage drop in the thyristor to the phase voltage (curve 3). This dependence is well approximated by below formula.

$$\frac{I_{2.0}}{I_{2.nom}} = 0.43\alpha \cdot e^{-0.46\alpha} \tag{12.23}$$

This last dependence in conjunction with the curve of Fig. 12.8 enables us to define the dependence of ignition angle and combustion angle of thyristors when network winding current passes through zero on the value of α:

0.043	0.2	0.4	0.6	0.8	1.0
$\underline{168}^0$	155°	143°	135°	131.4°	129°
$\underline{24}^0$	50°	74°	90°	97.2°	102°

Thus with increase in α, the ignition angle of thyristors when CSCT current passes through zero decreases and combustion angle of thyristors increases correspondingly.

The obtained result enables us to change the definition of angle characteristics of CSCT: dependence of CSCT current on the ignition angle of thyristors.

Let us bring the relative reactance of thyristor limb K in the earlier stated formulas of CSCT taking the base value of reactance when CSCT current passes through zero:

$$X_{2eq} = -KX_C = \frac{K}{\alpha}(X_{Lnom} + \alpha\delta X_{12}) \tag{12.24}$$

Thus we get the equivalent reactance of CSCT at any value of K in the form of:

$$X_{eq} = X_{Lnom}\frac{2K\sqrt{(1+\alpha-2\alpha\delta)^2 + 4\alpha\delta^2} - (1+\alpha-2\alpha\delta)}{2(1-K)\alpha\delta} \tag{12.25}$$

Consequently, the ratio of CSCT current to rated inductive current at any value of K

$$\frac{I_{(K)}}{I_{L.nom}} = \frac{X_{Lnom}}{X_{eq}} = \frac{2(1-K)\alpha\delta}{2\delta\cdot K + \sqrt{(1+\alpha-2\alpha\delta)^2 + 4\alpha\delta^2} - (1+\alpha-2\alpha\delta)} \tag{12.26}$$

Considering that the value δ does not influence the relative value of current, we take it to equal its minimal value $\delta = 0.5$.

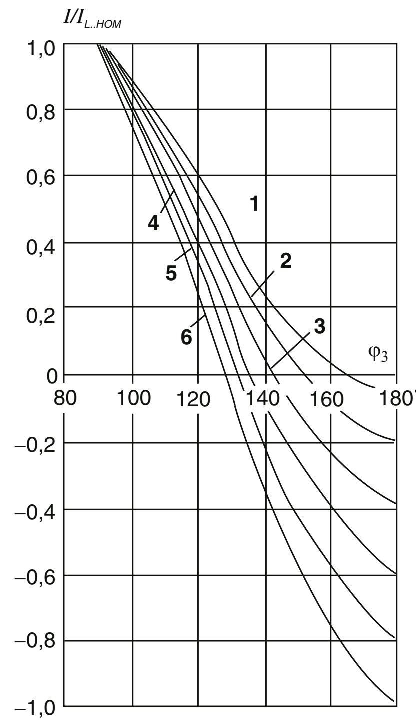

Fig. 12.9 Dependence of the relative value of current of CSCT on ignition angle of thyristors: 1 – $\alpha = 0.043$; 2–0.2; 3–0.4; 4–0.6; 5–0.8; 6–1.0

Then the relation (1.165) is substantially simplified

$$\frac{I_{(K)}}{I_{L.nom}} = \frac{(1-K)\,\alpha}{K+\sqrt{1+\alpha}-1} \tag{12.27}$$

Let's define the relative value of current of thyristors complying with the passing of network winding current through zero by the curve in Fig. 12.7. For example, when $\alpha = 1$ according to the above stated $\varphi_{ig} = 129°$ and correspondingly the value $I2/Imax = 0.3$. If the equivalent reactance of thyristor limb at constant voltage is halved ($K = 0.5$), the relative value of current through the thyristors will equal to $0.3/0.5 = 0.6$. Corresponding ignition angle of thyristors according to curve of Fig. 12.8 equals 110°. We define the relative value of CSCT current $I/Inom = 0.55$ by formula 12.27, therefore when $\alpha = 1$ that ratio corresponds with the angle $\varphi_{ig} = 110^0$. Thus all dependences from ILnom to ICnom (see Fig. 12.9) can be drawn.

12.3 Conclusion

CSCT as a new device to compensate reactive power in power systems was introduced in the paper. The main scheme of this device was also illustrated.

With a glance on the scheme of CSCT it seems to be the best way to add an additional capacitor to the ComW to have bilateral reactive power compensation. However with a mathematical analysis the idea can be changed to correct design of value of the harmonic filter elements in order to achieve a bilateral compensator can be obtained without essential need of capacitor bank.

In summarized, the paper endeavors to present final equations which are required to design the correct values of harmonic filters' elements to regulate of capacitive reactive power as well as the inductive value. The obtained results prove achieving this purpose.

References

1. G.N. Alexandrov, Stabilization of an operating voltage in electrical networks. News of Academy of Science. Energetic, 2001, W2, P. 68–79.
2. C. Gama, Y.C.S. Salomao, D. Gribek Yoao, Wo Ping. Brazilian north-south interconnection application of thyristor controlled series compensation (NCSC) to damp inter areas oscillation mode. The Future of Power Delivery in the 21st Century, 1997, Nov.
3. G.N. Alexandrov, Static thyristor compensator on the basis of a controlled shunt reactor of a transformer type. Electrichestvo, 2003, N22.
4. G.N. Alexandrov, Feature of the magnetic field of transformer under loading. Electrichestvo, 2003, N25, P. 19–26.