A Study on Probabilistic Evaluation of Harmonic Levels Produced by Static Compensators

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Abstract- **The probabilistic study of harmonics is becoming an important issue in power systems since the usage of power electronic devices is necessarily unavoidable. This has become more important due to the need for more reliable harmonic studies which consider the stochastic nature of produced harmonics in electrical power systems. This provides a probabilistic remedy that needs to be incorporated in the harmonic analyzes. This paper briefly reviews the related literature, and presents a tangible view on the probabilistic aspects, methods and standards of harmonics in electrical power systems. Then, a semi-stochastic method is proposed to predict and simulate the three-phase voltage unbalance, leading to an analytical tool for prediction of harmonic performance of STATCOM. This method is developed based on the measured data obtained from a low voltage distribution network. Finally, the modeled STATCOM is linked with the distribution substation, applying the proposed voltage unbalance modeling to evaluate uncharacteristic harmonics injected by the STATCOM at the PCC. This investigation on STATCOM illustrates the proper application of the probabilistic methods in practical and theoretical situations.**

I. INTRODUCTION

THE evaluation of harmonics in line with power quality assessment has become more important in electrical delivery systems. One of the main reasons is due to an increase in the use of power electronic devices employed to make power systems flexible and controllable. This can especially be considered in the deregulated environment from the viewpoint of both consumers and suppliers. In this context, developing countries, in particular, need to increase power transfer capacity in order to manage transmission congestion.

Hence, installation of power electronics static compensators would be inevitable for power systems. Consequently, harmonic penetration could be considerably an important concern on power quality of power systems because of operation of power electronic devices under high switching frequency compared to the power system frequency. Harmonic disturbances related to power electronic apparatuses have also become an important research topic for power system engineers and can no longer be ignored in industrial power systems since this might lead to problems such as capacitor failure or overheating of transformers and neutral conductors.

Study on harmonics involves in deterministic and probabilistic analysis. Deterministic criteria, however, ignores the variability of nonlinear loads and resultant changes in harmonic currents injected by these loads into the utility network. Also, increasing attention has been given to deterministic study on harmonics produced by several kinds of non-linear devices such as static var compensators, switching power converters and etc. These are subjected to random variations because of fluctuations in their operating points.

One common approach to conduct a deterministic study is based on the worst case in order to provide a safety margin in system design and operation. Nevertheless, this often leads to over-design and excessive costs. Consequently, statistical techniques for harmonic analysis are more suitable, similar to other conventional studies like probabilistic load flow and fault studies. Such analysis would calculate harmonic currents and voltages not simply derived from the expected average or maximum values. It would also obtain the complete spectrum of all probable values together with their respective probabilities.

This paper briefly reviews the background literature on the subject of applying probabilistic methods to the harmonics of static compensators in the following section. Furthermore, the shortcomings are noted related to the methods of probabilistic harmonic analysis and possible remediation options are proposed. Considerations and accuracy of practical implementations for probabilistic spectral analysis are thrashed out in section III. The presented discussion in section III leads to a conclusion about the use of Monte Carlo simulation. As a critical case study, the assessment of injected harmonics by an optimal-PWM-modulated STATCOM in interaction with the voltage unbalance of a real network is investigated in the remaining sections. This illustrates the usefulness and proper application of Monte Carlo simulation method as a probabilistic remedy in modeling practical harmonic injections.

II. BACKGROUND AND DISCUSSIONS

The probabilistic modeling and analysis of power system components harmonics goes back to the early 1970s [1]. This involves in a mathematical analysis which is used on instantaneous values of currents from individual harmonic components. One of the first attempts has been carried out by Rowe [1] in which he considers the addition of a series of currents modeled as phasors of random amplitudes and random phase angles. The current vectors are assumed to be produced from a group of distorted loads. Rowe's analysis is limited to the derivation of the resulted current properties. Two further assumptions also have been made using a uniform probability density function; first, the amplitude is variable from zero to a peak value; second, the phase angle is variable from 0 to 2π. Properties of the summation current are obtained by simplifying the analysis by means of a Rayleigh distribution function when the number of random vectors is large enough.

Pierrat [2] has extended the work of Rowe to form a general case by revising the phase angle of a phasor to be uniformly distributed in the range of $[-\varphi, \varphi]$, $0<\varphi<\pi$. He has proved that the Rayleigh distribution obtained by Rowe is only a special case when φ equals π. It should be noted that the magnitude and the phase angle of each harmonic vector in [1] and [2] have been assumed to be independent. Since the real part and the imaginary parts of harmonic currents of the same order are respectively summed up, the sum of the two resolved components would approach to a normal distribution function when the number of random vectors is sufficiently large [2].

A bivariate normal distribution (BND) model has been applied to some studied cases such as [3, 4]. Five parameters have been employed by Kazibwe, et al. [3], namely, the mean values and variances of the two resolved sums and their correlation coefficient of BND model which measures the dependence between the harmonics vectors components.

In a later work, the same BND model has been employed to determine the magnitude of harmonic contents generated by ten independent power converters in a distribution system, where Monte Carlo simulation has been performed to verify their theoretical results [5]. The BND approach is also used to predict voltage distortion in a distributed power system. The analyzed results are examined using both simulation and field measurements [6].

Probabilistic schemes have been presented in [7] for studying harmonic currents produced by an AC-to-DC converter under unbalanced voltages. In [8], it is compared randomly generated harmonic currents produced by power converters using BND and direct normal distribution (DND). Probabilistic models for studying harmonic currents generated by 12-pulse AC-to-DC power converters have been proposed in [9], while harmonic impacts on the distribution systems have been demonstrated in [10] using a probabilistic scheme.

The early analyzes assume various limitations and PDFs for some parameters such as firing angles of converters. They consider simple models in order to achieve applicability in spite of lower accuracy for harmonics evaluation. Additionally, Morrison and Clark have measured the actual probability density functions for the currents of a substation fed from AC traction supplies of a metro rail [11]. They have found that the actual measured PDF differs from the simulated one. Using more realistic simulation of PDFs, simulated statistical values of summation current is estimated for the substation currents with some good agreement between estimated and measured cumulative distribution functions (CDFs).

Meanwhile, there exist both theoretical and practical issues to be resolved. Several factors and shortcomings were noted related to the methods of probabilistic harmonic analysis as follows:

- There is a lack of knowledge concerning the actual PDFs for almost a majority of loads.
- For a certain load, it is complicated to relate mutually between different harmonic currents. It should be noted that it is also somehow unknown the degree of independence of different harmonic currents from each other.
- Load arrangement as well as the voltage supply is constantly subjected to changes on a feeder.
- There are a large number of nonlinear loads in the system at any given time. Nonlinear loads respond to voltage deviations in different ways. In other words, there is a lack of understanding on how different voltage waveforms together with output power levels affect the harmonic currents (e.g. static compensators).
- Generally, load models as well as the system Thevenin impedance are not fully known at various frequencies.

The mentioned issues cause some sort of reluctance to fully believe in probabilistic analysis of harmonics. Since the waveforms are subjected to changes rather than certain steady state harmonic decomposition, it might also be raised question on the validity probabilistic method. Nevertheless, it can be reasonably assumed that the power network variations are inherently slow (i.e. quasi steady state), which allows utilizing probabilistic methods with analytical confidence.

It has been recognized that probability theory can be especially helpful when attempting to determine compatibility levels for low-frequency harmonic distortion [12]. However, the non-stationary nature of harmonics requires detailed analysis and accurate measurements. Meanwhile, the use of probabilistic methods applied to spectral analysis may not conform well to all conditions, particularly when the rate of change of waveforms is fast. Therefore, the subject demands a comprehensive analytical understanding, study on its validity, and research on new techniques to overcome the analytical issues.

III. CONSIDERATIONS AND ACCURACY OF PRACTICAL IMPLEMENTATIONS

Harmonic analysis might be applied only when waveforms of currents and voltages are periodic, operating in a certain steady-state for some time. The most popular technique used for harmonic calculations is the use of time-domain formulation. Harmonic information is then obtained using the fast Fourier transform (FFT) in steady state [16]. This technique gives very accurate results when the signal is strictly periodic, the sampling frequency is an integer multiple of the fundamental frequency, and the sample frequency is at least twice the highest frequency in the signal (according to the sampling theorem). Assuming the mains period is fixed, voltages and currents waveforms are constantly changing with time depending on the load dynamic. Thus, time-axis of the frequency spectral are generated by short-time Fourier transform (STFT), where each time-frequency window correspondingly relates to the magnitudes of the obtained spectra. The drawback is that different window sizes give different spectra since deviations often exist within any selected window size. As a consequence, several errors can occur when calculating harmonic levels by direct application of windowed FFT.

Other problems attached to this technique are the difficulty of describing frequency-dependent parameters [17], and the time-domain formulation requires considerable computation steps even for relatively small systems [18]. The problem of applying probabilistic methods to stochastic harmonic distortion should be revived and revised.

On the other hand, the non-linear loads such as static compensators could also interact with possible harmonic distortions and unbalances of the power network (e.g. distributed systems are coined with many power electronic loads). These interactions would be complex and uncharacteristic, making the analysis and assessment of harmonic levels non-deterministic. This uncertainty could be impressive in some situations such as penetration of an optimal-PWM-modulated STATCOM. In such a case, the compensator uses pre-calculated switching angles based on assuming an ideal fixed DC bus voltage in order to eliminate undesirable low-order harmonics. On the other hand, load-terminal harmonics and unbalance of the distribution system impose distortion on both DC and AC sides [16, 19], making the pre-calculated chopping angles non-optimal [16]. This introduces additional harmonics generated by the STATCOM which are uncharacteristic considering the OPWM selective harmonic elimination. Hence, the amount of uncharacteristic harmonics that is injected to the distribution system depends on several factors such as ratings and operating conditions of both STATCOM and distribution network. Nonetheless, it is always probable that the magnitudes of these harmonics exceed the normal value which should be taken into account in some situations such as multiple penetrations of power electronic devices and in order to achieve more precise evaluation and modeling of harmonics.

One useful approach focuses on the concept of evolutionary spectrum applied to time-varying harmonic distortion. This method can help the power quality engineer to gain a better understanding of the nature of such variations as well as properly utilizing analytical tools to predict their behavior. The investigations and applications should include practical waveforms together with field measurements. This is necessary to validate the evolutionary approach.

A different problem arises when we assume a number of nonlinear loads (e.g. power electronic converters) are supplied from a given feeder bus. The harmonic current at a certain order can be worked out by complex summation of that harmonic number absorbed by each load. Hence, one needs to obtain both the magnitude and phase angle (with respect to a common reference) of each load for a given harmonic number in order to perform these calculations. Obviously, the sum of the harmonic currents at each order for all distributed harmonic sources, if assumed to have some statistical variations generally is less than the arithmetic sum of the maximum values. In order to deal with the computational problem of such a sum, the harmonics phasors should be represented in terms of its joint probability density function (jpdf) and its rectangular components in terms of their marginal probability density function (pdf). Because of complexity of conditional probabilities used to describe dependent harmonic phasors, all papers to date [1-11], have only studied independent harmonic sources. Each jpdf is characterized by six parameters; the mean value and standard deviation of both x and y, covariance*,* and correlation coefficient. The jpdf of the summation harmonic is obtained analytically by convolution (when the variables are independent) or by using the Fourier transform technique. Nonetheless, evaluation by both methods is difficult to achieve compared to the Monte Carlo as an alternative simulation method [13-15].

In practice, accurate analytical expressions (describing joint probability density functions of harmonics) are not easy to obtain from scatter plots in the x-y plane. This is due to the fact that actual distribution functions are often spread over complex surfaces; especially when a deterministic component is present.

Furthermore, more often only harmonic current magnitudes are measured and their corresponding phase angles are unknown. In such situations, assuming that these current are in phase will surely lead to conservative results. On the other hand, assuming a uniform distribution over the entire phase angle range may lead to a total current that is lower than the actual value. Anyhow, there is a tangible need for analyzes in which these simplifications are avoided as much as possible while maintaining applicability of the method.

It is often desired to determine the characteristics of harmonic voltages resulting from random harmonic current injections at multiple nodes within a utility network. In practical systems, the voltage distortion is rarely above few percents. Thus, one can assume that injected harmonic currents by nonlinear loads are not affected by the resulting harmonic voltages. In such a case, the harmonic voltage of order h is simply computed by superposition.

However, analytical relationships are required to describe the impact of a change in harmonic voltage on the injected current when this dependency is not negligible. Unfortunately such knowledge is limited to very few specific non-linear loads. Up to date, the studied cases, except simple ones [2-12], have been considered too complex to study analytically such dependencies and they have resorted to either the central limit theorem when evaluating the sum of a number of random harmonics, or the Monte Carlo simulation which can handle practically any statistical problem [12]. Meanwhile, the Monte Carlo simulation method is favorable over analytical methods because of its simplicity. Thousands of simulations are often required, while this is not a computational burden in nowadays super-fast microprocessor.

IV. Detailed Study of STATCOM Harmonics

The static synchronous compensator (STATCOM) is

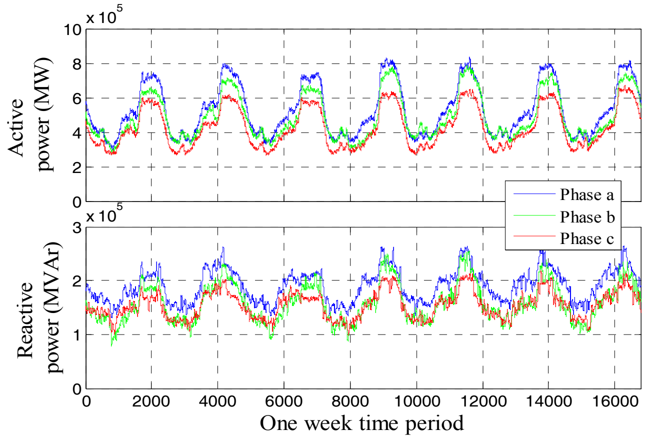

Figure 1. Recorded three-phase active and reactive powers by data logger that is installed at 400 V substation of Mavad.

increasingly used in power systems in order to increase power transfer capability, provide voltage support, and generally improve power quality in power distribution systems. In this section, the harmonics of STATCOM is discussed from a probabilistic viewpoint. The presented preliminary analysis is intended to illustrate the proper application and quiddity of the probabilistic methods in such cases.

Over the last years, an increasing amount of research work addressing the STATCOM has been published in the open literature mainly on design, operation and control issues [20]. Various modulation techniques and topologies are suggested for STATCOM to effectively remove the generated low-order characteristic harmonics, including multi-module PWM techniques, selective harmonic elimination and multilevel topologies [21], [22]. An optimal pulse width modulation (OPWM) uses pre-calculated switching angles based on assuming an ideal fixed DC bus voltage. Like other PWM schemes, the DC-link voltage is modulated by the converter using the programmed harmonic elimination technique, i.e. the OPWM. Both switching instants and durations are pre-calculated off-line such that certain chosen harmonics are eliminated; also, a desired value is assigned to the fundamental component of the output. Normally, a set of solutions for a certain range of fundamental voltage magnitudes is pre-calculated and stored in a look-up table for on-line implementations. Detailed discussions on implementing the OPWM can be found in [25].

This method presents several advantages in comparison to the conventional carrier-modulated sinusoidal PWM schemes [23]. However, as stated in the section III, the use of this technique may leads to the generation of uncharacteristic harmonics in realistic operating conditions [16, 24].

Therefore, the following steps should be performed to obtain a more reliable harmonic performance evaluation of STATCOM:

- Developing a comprehensive model which takes into account relevant factors and interactions in harmonic generation such as DC capacitor oscillations, capacitance limitation, and the grid system unbalance. This model would provide accurate and efficient analytical capability.
- Simulating a realistic distribution system under unbalance and distorted condition. The environment should be also capable of integrating deterministic and stochastic approaches, if necessary.
- Making the simulation components efficient and fast to avoid unnecessary iterative procedures that may slow down complementary simulations such as Monte Carlo method.

The first step is a challenging topic that has devoted a noticeable amount of research work over the last few years. Generally speaking, there are three philosophies in modeling of devices, network and their interactions. The methods are embedded in time-domain, frequency-domain, and harmonic-domain. In this section, the harmonic-domain model of [24] is used to tackle with the problems which arise when the solutions are embedded in the time-domain. These problems are briefly discussed in section III. Monte Carlo simulation is then be applicable with the aid of this harmonic-domain model which provides fast calculations.

The second and third steps should be considered and optimized because of probabilistic nature of the generated and the background harmonics. This paper proposes a detailed semi-stochastic modeling at the PCC for the voltage unbalance. The realistic modeling of voltage unbalance is provided because it affects considerably the harmonic performance of the STATCOM. Then, this realistic representation of the PCC is applied to the model of the STATCOM. Resultant outcomes show the penetrated THD of OPWM-STATCOM AC current due to the uncharacteristic harmonics. This can be easily performed with other modulation techniques under a realistic operating condition. Meanwhile, a real case study is arranged in which the voltage unbalance and background harmonics at a distribution substation are simulated. This substation is chosen from part of Tehran north-east distribution system. Also, a STATCOM will be connected across this real substation. It is eventually examined uncharacteristic harmonics produced by STATCOM that is subjected to the interaction with the realistic model of the PCC.

V. Modeling and Simulation of System Unbalance

Operation of STATCOM under three-phase unbalanced voltages affects noticeably the penetration of uncharacteristic harmonics which are not usually targeted by the engaged modulation technique [2].

A. Case study: System Description and Measurements

A data logger is installed at the distribution substation of Mavad that is located in north-east of Tehran. The measured data is gathered during a week in September 2002. Fig. 1 demonstrates the recorded active and reactive powers of the three phases at this substation for one week.

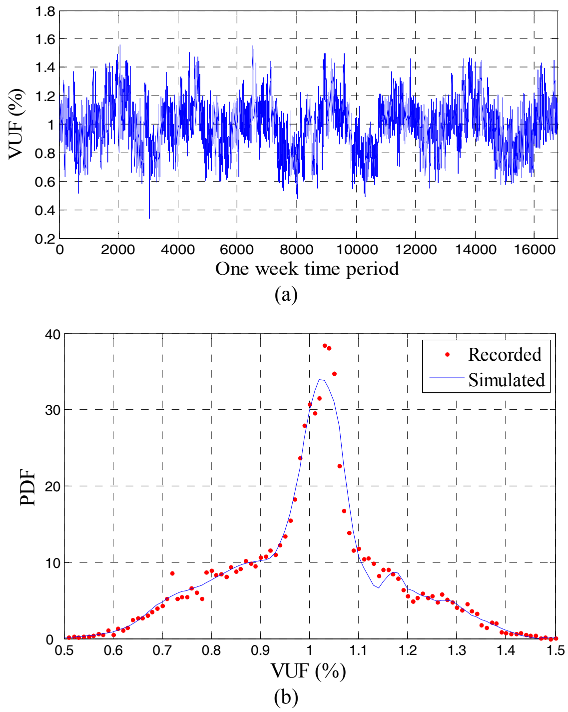

Figure 2. (a) The VUF% variation over the one week period at the low-voltage side of T3 obtained using the proposed simulation method, and (b) comparing the PDFs of VUF% obtained from field measurements (dotted line) and the proposed algorithm (solid line).

B. Voltage Unbalance Simulation

The following procedure is suggested to be used here for simulating voltage unbalance:

1) Active and reactive powers are split into two parts: *probabilistic* and *deterministic* components. This is performed using the *wavelet* transform. Also, the analysis is managed for two categories of *working days* and *week-end* separately like those of the load forecasting algorithms.
2) The *approximation* parts (obtained from the wavelet transform) are then directly predicted as the *average daily power* curve.
3) The *detail* parts of the wavelet transform are modeled using a *copula* which generates six correlated random variables corresponding to *active and reactive powers* of the three phases.
4) Time-domain active and reactive powers (modeled during the previous steps) are reconstructed by applying the inverse wavelet transform.
5) The reconstructed powers are then used to predict the *three-phase voltages* through a three-phase load flow program using the *Monte Carlo* simulation. The IEC voltage unbalance factor (VUF) is calculated from the obtained voltages in step 4.

This procedure provides a more realistic insight into the VUF at the PCC. It is also compatible with the Monte Carlo simulation program and statistical evaluations. Wavelet transform presents suitable filtering characteristic. This enables the procedure to introduce the power as a deterministic component (approximation parts) with a quite stable mean and standard variation. Also, the procedure introduces a nearly Gaussian distributed probabilistic component (detail parts). Assuming the Daubechies (db5) mother-wavelet, up to the fourth filtering level is used to determine approximations and details. It is noticeable that the probabilistic component can be modeled using a Gaussian *copula* [26, 27].

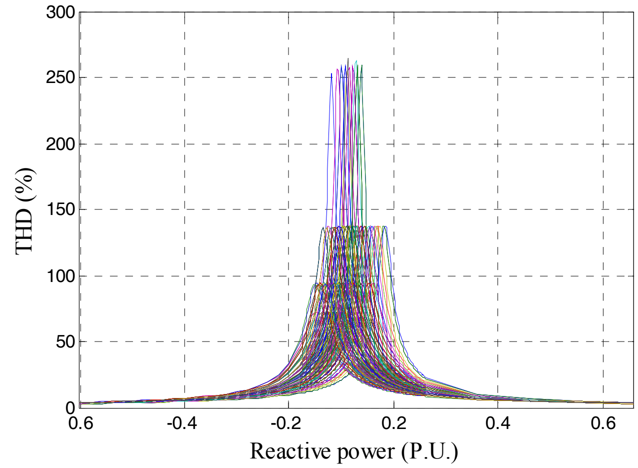

Figure 3. Penetrated THDS due to the 5th, 7th, 11th, 13th, 17th and 19th harmonics under various VUF% and reactive power obtained from Monte Carlo Simulation.

Simulations are shown in Fig. 2 during a week, varying within [0.38%, 1.58%] for the distribution substation (Mavad). The voltage unbalance percent (VUF%) of the substation of Mavad is calculated at the low-voltage side (400 V) of the transformer T3 using MATLAB. To validate the suggested algorithm, the probability distribution functions are obtained from both the proposed unbalance algorithm and field measurements. Both PDFs are shown in Fig. 2(b). Comparing the exact collected data (dotted line) with those of the proposed method (solid line), it can be seen that the suggested algorithm provides an accurate simulation of unbalance variation at the PCC.

VI. STATCOM HARMONIC PERFORMANCE UNDER UNBALANCE OPERATION

A realistic voltage unbalance case is studied using the foregoing suggestion at a distribution substation. It is also shown that the distribution system may operate under certain amounts of voltage unbalance (see Fig. 2(a)). It can now be studied the effects of voltage unbalance on the harmonic performance of the OPWM-STATCOM. The harmonic-domain model of [24] for STATCOM is linked with the Monte Carlo simulation of the calculated unbalance percent (VUF%) in Fig. 2(b).

Assume the capacitance of the DC-link capacitor of STATCOM is 2 mF. Then, the resultant THD% is calculated for all data as shown in Fig. 3. Background harmonics are represented by voltage harmonic source obtained from field measurements. Each curve represents a trace of THD variation over various reactive power loadings. Various curves are calculated for each probable VUF which are realistically modeled by the proposed procedure in the previous section.

It can be seen from Fig. 3 that a realistic voltage unbalance would not dramatically modifies the uncharacteristic THD of STATCOM AC current (THDS) except for situations that STATCOM absorbs relatively small amounts of reactive power. As an example, consider the cases when four different reactive powers are supplied by STATCOM. It is observed from Figs. 3 that reactive powers of -0.6 P.U. and 0.6 P.U. along with realistic VUF% give a relatively small range of the THDS variations around 5%. However, reactive powers between -0.1 and 0.2 P.U. along with the same realistic VUF% result in a considerable range of the THDS variations as high as 140% and even rarely higher. It should be noted that the number of points for each amount of reactive power over the THDS curves, represents the probability of occurrence for possible THDSs.

It should also be noted that the THDS is affected by changing the DC-link capacitance. Also, the power system equivalent impedance influences the THDS under variation of the voltage unbalance percent. This analysis can be easily extended to include other realistic conditions. It is theoretically possible to modify the OPWM procedure to remove uncharacteristic harmonics under unbalanced conditions. However, the analysis become more complex, it is not purely deterministic and could be difficult to implement online.

VII. Conclusion

A brief overview and better understanding of the probabilistic aspects of harmonics in electrical power systems is inspected in this paper. It addressed questions on the previous literature, accuracy of collected data and probabilistic methods with some practical considerations, issues of summing harmonic phasors, and determining the statistical characteristics of harmonic voltages resulting from multiple harmonic current injections in a power network. The more stress is on harmonics of the static power converters. It has been determined that analytical solutions can only be derived for basic and simple cases. For practical situations, the Monte Carlo simulation method is still the best choice to date. However, analytical methods are always indispensable to the interpretation of results obtained from the simulation. Also, there must be a consideration on the mechanism of harmonic generation (deterministic model) in nonlinear loads such as static converters that presented here in performing the probabilistic solution correctly.

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