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Security-constrained transmission expansion planning: A stochastic multi-objective approach

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ABSTRACT

This article presents a stochastic multi-objective optimization framework for transmission expansion planning (TEP) with steady state voltage security management, using AC optimal power flow (AC-OPF). The objectives are to minimize the sum of transmission investment costs (ICs), minimize the Expected Operation Cost (EOC), minimize the Expected Load Shedding Cost (ELSC) and maximize the Expected Loading Factor (ELF). The system load uncertainty has been considered and the corresponding scenarios are generated employing the Monte Carlo (MC) simulations. A scenario reduction technique is applied to reduce the number of scenarios. A multi-objective mathematical programming (MMP) is formulated and the ε-constraint method is used to solve the formulated problem. The $N-1$ contingency analysis is also considered for the proposed TEP problem.

The proposed TEP model has been applied to the well-known IEEE 24-bus Reliability Test System. The detailed results of the case study are presented and thoroughly analyzed. The obtained TEP results show the efficiency of the proposed algorithm.

1. Introduction

Transmission expansion planning (TEP) addresses the problem of augmenting an existing generation and transmission network to optimally serve a growing electric load while satisfying a set of economical, technical and reliability constraints [1]. The TEP responds to the problem of where, when and what type of new lines and transformers should be built with minimum costs in a particular planning time period meeting some constraints and criteria.

Based on the planning horizon there are two types of TEP, (1) static transmission expansion planning (STEP) and (2) dynamic transmission expansion planning (DTEP). The DTEP determines when the new lines should be installed. The expansion planning of transmission can also be categorized into short-term, mid-term and long-term planning based on the time horizon. Short-term has a time horizon of 3–5 years. This should be contrasted with medium/long-term expansion plans where time horizon can extend up to 30 years [2].

Naturally the TEP problem is a non-convex, non-linear large scale optimization problem, which is difficult to solve. Based on the solution methods, the TEP can be classified into three types: (1) mathematical optimization, (2) heuristic methods and, (3) meta-heuristic methods. Ref. [3] is a good paper that classifies the publications on the TEP problem and reviews the models and articles in this area.

A mixed integer linear programming that considers losses is used in Ref. [4]. Roh et al. [5] presented a stochastic coordination of generation and transmission expansion planning model in a competitive electricity market. The authors of Ref. [6] presented a bi-level optimization model for transmission expansion planning within a market environment, where the suppliers and consumers traded electric energy through a Day-Ahead market. Torre et al. [7] employed a mixed-integer linear programming (LP) formulation for the long-term transmission expansion planning problem in a competitive pool-based electricity market. The above mentioned articles used the DC power flow in order to solve their formulated TEP problem, but in this paper an AC power flow based security constrained TEP (SC-TEP) is formulated that also considers the static voltage security as an objective in its optimization process.

The effects of inflation rate and load growth on the TEP problem have been investigated in Ref. [8]. Ref. [9] has analyzed the TEP and Generation Expansion Planning (GEP) problem together. The authors in [10] have modeled a multi-stage stochastic TEP problem including available transfer capability (ATC). They showed involving this criterion will increase the ATC between source and sink points and as a result power system reliability will be increased and more money can be saved. A transmission expansion planning

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Nomenclature

Indices

c	index of $N-1$ contingency (single line outage) criteria, $c = 0$ is related to the normal conditions
g	index of generators
i, j	indices of buses
l	index of lines
s	index of scenarios
t	index of time periods

Sets

B	set of all buses
C	set of all (single) contingencies
CL	set of all candidate lines
G	set of all generators
EL	set of all existing lines
S	set of all scenarios
Y	set of all time periods

Constants

$AP_{L,l}^{\max}$	maximum apparent power flow of line l
C_{2g}, C_{1g}, C_{0g}	quadratic offer function coefficients, i.e. $Offer_g = C_{2g}P_{G,g}^2 + C_{1g}P_{G,g} + C_{0g}$
I	annual (discount) interest rate
IC_l	investment cost of candidate line l
nb	number of buses
nc	number of candidate lines
ne	number of existing lines
ng	number of generators
nl	number of total lines (existing and selected candidate lines)
N_l	limits on the number of lines constructed between two buses
ns	number of scenarios
$P_{D,ist}$	active power demand at bus i for scenario s in time period t
$P_{G,g}^{\min}$	minimum active power of generator g
$P_{G,g}^{\max}$	maximum active power of generator g
pf_i	load power factor at bus i
$Q_{D,ist}$	reactive power demand at bus i for scenario s in time period t
$Q_{G,g}^{\min}$	minimum reactive power of generator g
$Q_{G,g}^{\max}$	maximum reactive power of generator g
$rp_i^{\max}$	maximum amount of active power load shedding allowed at bus i
$rq_i^{\max}$	maximum amount of reactive power load shedding allowed at bus i
T	number of time periods
TD	time duration of time period t (in year)
$\lvert V_i^{\min}\rvert$	minimum voltage magnitude at bus i
$\lvert V_i^{\max}\rvert$	maximum voltage magnitude at bus i
$Y_{ij}^0 = G_{ij}^0 + jB_{ij}^0, Y_{ij} = G_{ij} + jB_{ij}$	admittance of line ij for the existing and candidate lines, respectively
α_i, β_i	penalty factor for active and reactive power load shedding at bus i
π_s	probability of scenario s
θ_{ref}	reference phase angle for the slack bus

Variables

ELF	Expected Loading Factor
ELSC	Expected Load Shedding Cost
EOC	Expected Operation Cost
IC	Total transmission investment cost
K_G	a variable to model the distributed slack bus
LF_{st}^c	loading factor in time period t for scenario s and contingency c
$P_{G,gst}^c, \widetilde{P}_{G,gst}^c$	active power of generator g in time period t for scenario s and contingency c for the current operating and maximum loading points, respectively
$P_{L,lst}^{0,c}, \widetilde{P}_{L,lst}^{0,c}$	active power flow of existing line l in time period t for scenario s and contingency c from bus i to bus j for the current operating and maximum loading points, respectively
$P_{L,lst}^c, \widetilde{P}_{L,lst}^c$	active power flow of candidate line l in time period t for scenario s and contingency c from bus i to bus j for the current operating and maximum loading points, respectively
$Q_{G,gst}^c$, $\widetilde{Q}_{G,gst}^c$	reactive power of generator g in time period t for scenario s and contingency c for the current operating and maximum loading points, respectively
$Q_{L,lst}^{0,c}$, $\widetilde{Q}_{L,lst}^{0,c}$	reactive power flow of existing line l in time period t for scenario s and contingency c from bus i to bus j for the current operating and maximum loading points, respectively
$Q_{L,lst}^c$, $\widetilde{Q}_{L,lst}^c$	reactive power flow of candidate line l in time period t for scenario s and contingency c from bus i to bus j for the current operating and maximum loading points, respectively
rp_{ist}^c	amount of active power load shedding at bus i for scenario s and contingency c in time period t
rq_{ist}^c	amount of reactive power load shedding at bus i for scenario s and contingency c in time period t
u_{lt}	binary variable: 1 if line l is constructed in time period t, 0 otherwise
$\lvert V_{ist}^c\rvert$, $\lvert\widetilde{V}_{ist}^c\rvert$	voltage magnitude at bus i in time period t for scenario s and contingency c for the current operating and maximum loading points, respectively
θ_{ist}^c, $\widetilde{\theta}_{ist}^c$	voltage phase angle at bus i for scenario s and contingency c in time period t for the current operating and maximum loading points, respectively

was formulated by Sadegheih and Drake [11] and solved for using mixed integer programming, genetic algorithm (GA) and tabu search (TS). They showed that the GA algorithm was more successful in finding the optimal TEP solution and could avoid local optimums. Ref. [12] studied TEP considering the load uncertainty using benders decomposition. Al-Hamouz and Al-Faraj [13] considered the cost of ohmic and corona losses together as an objective function in addition to the cost of investment for constructing the new transmission lines. Wu et al. [14] suggested a framework to clarify the interactions among various economic and engineering issues by reviewing recent theoretical and practical progresses in transmission investment and transmission planning methodology. Alguacil et al. [15] proposed the reinforcement and expansion of the transmission network as a way of mitigating the impact of increasingly plausible deliberate outages. The benefits of transmission expansions in competitive electricity markets have been investigated in Ref. [16]. They presented an innovative method for assessing simultaneously the technical and economical benefits of transmission expansions. Ref. [17] evaluated transmission plans under deterministic or uncertain condition and concluded that Transmission plans under uncertainty performed better than deterministic plans. A transmission planning model under a deregulated market was presented in [18,19]. Ref. [20] proposes a new methodology to solve transmission expansion planning (TEP) problems in power system, based on the metaheuristic ant colony optimization (ACO).

Almost all of the papers published about TEP have used DC models for power flow analysis, which are not completely suitable due to ignoring the voltage security and reactive power issues in the TEP problem. Although Ref. [21] uses AC-OPF for transmission planning, however does not consider the voltage security issues in the power system. Using AC-OPF could provide us a more precise picture of the active and reactive power flows in the expanded power network and also show us how to plan for maintaining the adequate voltage stability margins throughout the network. Therefore, in this paper we introduce the Voltage Stability Margin (VSM) as one of our objective functions for maintaining the adequate security of power system under normal conditions as well as contingency conditions during the planning horizon using an AC-OPF approach. The proposed TEP mathematical model is novel and has not been presented in the literature before. In this paper the load uncertainty is considered, and a normal Probability Distribution Function (PDF) is assigned to the load of each bus with 5% annual growth rate in its expected value. It is assumed that the generators' supply offers are deterministic and known parameters to the Independent System Operator/Transmission System Operator (ISO/TSO). The method presented here is based on scenario tree construction. Also, it is assumed that the ISO/TSO is responsible for transmission expansion planning, and one of its most important responsibilities is to maintain the voltage security of the power system.

The rest of the paper is organized as follows: Section 2 presents the Voltage Stability Margin (VSM) definition and calculation. Section 3 presents the problem formulation of the transmission expansion planning with steady state voltage stability margin taking into account. The case study and its results are presented in Section 4, and finally the major contributions and conclusions are drawn in Section 5.

2. Voltage stability margin

One of the most important problems that recently received major attention is the voltage instability. Reports show the voltage collapse for the recent incident in North American power system on August 14th, 2003 caused approximately 50 million customers spend more than 15 h without electricity [22]. Furthermore, in the new open-access and competitive environment power systems are more heavily loaded than before because of the growing demands, maximum economic benefits and efficiency of usage of transmission capacity that have led to a higher probability of voltage instability or collapse [23], hence voltage stability assessment is an essential issue for power system planners and operators.

2.1. Definition

Voltage Stability Margin (VSM) is defined as the amount of additional load on a specified pattern of load increase that would cause power system instability. Fig. 1 shows a typical PV curve diagram for given status of power system which demonstrates the voltage variation versus active power demand. The difference between the current operating point and the voltage collapse point (distance between points A and B as shown in Fig. 1) from which

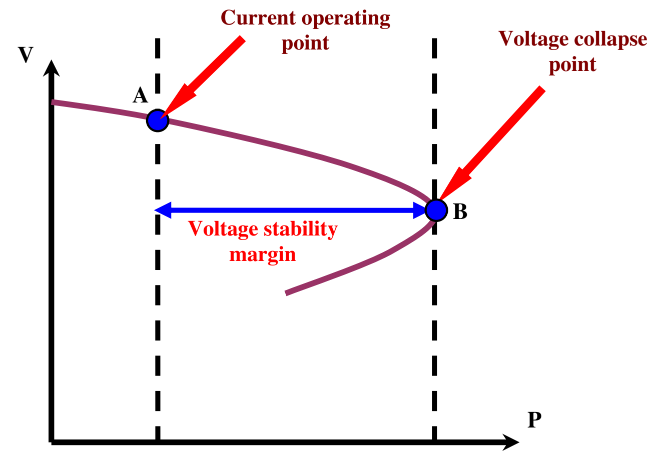

Fig. 1. A typical PV curve for a given condition.

power system loses stability is measured as the VSM. The Western Electricity Coordinating Council (WECC) proposes a minimum *P–V* margin requirement of 5% for a single contingency, 2.5% for double contingencies, and larger than zero for multiple contingencies ($N \geqslant 3$) [24].

2.2. Calculation

This margin can be obtained in a variety of ways. Voltage stability is indeed a dynamic phenomenon and can be studied using extended transient/midterm stability simulations [25]. Nevertheless, static methods have been used to a large extent in the literature, because they can provide suitable results with less computational burden. Gao et al. [25] present a method for voltage stability evaluation using modal analysis. In [26] authors use continuation power flow (CPF) in order to determine the VSM. A nonlinear programming method has been presented in [27]. Readers can refer to the pertinent available papers published in this area of research for more details and discussions.

Power system voltage security can be included in the transmission expansion planning problem by solving the following AC-OPF problem:

$$\max \ LF \tag{1}$$

$$\text{s.t.:} \ P_{G,i}(1+LF+K_G) - P_{D,i}(1+LF) = \sum_{l\in EL} P_{L,l} \quad \forall i \in B, \forall l \in (i,j) \tag{2}$$

$$Q_{G,i} - Q_{D,i}(1+LF) = \sum_{l\in EL} Q_{L,l} \quad \forall i \in B, \forall l \in (i,j) \tag{3}$$

$$P_{L,l} = |V_i|^2 G_{ij} - |V_i||V_j| \times (G_{ij}\cos(\theta_i-\theta_j) + B_{ij}\sin(\theta_i-\theta_j)) \quad \forall l \in EL, l \in (i,j) \tag{4}$$

$$Q_{L,l} = -|V_i|^2 B_{ij} - |V_i||V_j| \times (G_{ij}\sin(\theta_i-\theta_j) - B_{ij}\cos(\theta_i-\theta_j)) \quad \forall l \in EL, l \in (i,j) \tag{5}$$

$$P^{\min}_{G,g} \leqslant P_{G,g}(1+LF+K_G) \leqslant P^{\max}_{G,g} \quad \forall g \in G \tag{6}$$

$$Q^{\min}_{G,g} \leqslant Q_{G,g} \leqslant Q^{\max}_{G,g} \quad \forall g \in G \tag{7}$$

$$(P_{L,l})^2 + (Q_{L,l})^2 \leqslant \left(AP^{\max}_{L,l}\right)^2 \quad \forall l \in EL \tag{8}$$

$$\left|V^{\min}_i\right| \leqslant |V_i| \leqslant \left|V^{\max}_i\right| \quad \forall i \in B \tag{9}$$

The objective function in (1) denotes the loading factor. In the above Eqs. (2) and (3) are active and reactive power balance in each bus, where K_G is used to model a distributed slack bus. Active and reactive power flows at line l have been shown by Eqs. (4) and (5). Eqs. (6) and (7) represent limits on active and reactive power generation of units. Constraint (8) indicates the thermal limit of line l, while Eq. (9) constrains all bus voltage to be within appropriate limits.

The voltage security margin is implemented in the TEP problem by introducing the loading factor into the optimization problem that is presented in Section 3.2.

3. Multi-stage stochastic programming

A stochastic program is a mathematical program in which the uncertainties are shown as random variables with known probability distribution functions.

This section is arranged as follows: a brief review of the scenario tree construction and reduction techniques are presented in Section 3.1; then the problem formulation is given in Section 3.2.

3.1. Scenario tree construction and scenario reduction techniques

One way in stochastic programming is to replace the random variables by their expectations and then solve the deterministic mathematical programs. Another way is to consider all scenarios with the corresponding probabilities and then select the best plan among all scenarios.

Many approaches that are available for scenario generation are reviewed in Ref. [28]. The Monte Carlo simulation is applied in this paper for generating different scenarios. The Monte Carlo (MC) method is based on the repeated random sampling and statistical analysis of the simulation results. In the MC simulations after identifying the PDF of the input variables, some random samples are generated by the random number generator (RNG) and the output values are calculated in a deterministic model. This process is repeated many times until adequate numbers of output variables are produced. In this paper, it is assumed that the normal PDF of the system loads are available at all buses over the planning horizon. A scenario tree is represented by the finite number of nodes. It starts from a root node and finally terminates to the leaf nodes. Each node in scenario tree has an individual predecessor node, but presumably several successors. Each path from the root node to the leaf nodes is defined as a scenario. Due to the computational complexities and time limitations, the scenario reduction techniques are applied in this paper for reducing the number of scenarios by deleting the scenarios with small probabilities or bundling similar scenarios based on the method of Ref. [28]. The reduction algorithms determine a subset of the initial scenarios and assign new probabilities to the preserved scenarios.

3.2. The proposed SC-TEP problem formulation

Four objective functions are considered for the proposed SC-TEP problem as explained below:

(1) Investment Cost (IC), which is the construction cost of new lines and transformers.
(2) Expected Operation Cost (EOC), which is the expected cost of generation in the power system.
(3) Expected Load Shedding Cost (ELSC), which is the expected cost of load curtailment in the power system.
(4) Expected Loading Factor (ELF), which must be maximized for ensuring the adequate power system security.

Other objectives such as standby cost [1]; security cost [29]; loss cost [30] and congestion cost [31] are not considered in this paper, but the model could be easily extended to include these objectives as well. From the power system security point of view more objectives such as voltage drop or overload of the lines could be added to our formulated problem as well.

The objective functions that have been defined above can be conflicting with each other. Hence, a multi-objective approach is necessary when solving the TEP problem. In a multi-objective optimization, unlike single-objective optimization, there will be several conflicting objectives simultaneously. In such a case, there is usually no single optimal solution, but a set of alternatives with different trade-offs, called Pareto optimal solutions, or non-dominated solutions. Thus, a decision-making task will be essential in multi-objective optimization in order to choose a single most preferred solution, in addition to the optimization task for finding the Pareto optimal solutions [32]. A multi-objective framework for transmission expansion planning has been presented in [33] that considered the investment cost, reliability and congestion costs in the optimization as three objectives. Two of the most popular techniques to solve the Multi-Objective Mathematical Programming (MMP) problems (that have been widely used by researchers and planners) are the weighting method and ε-constraint method [32,34,35].

In order to properly apply the ε-constraint method, the ranges of at least $k-1$ objective functions are needed to be used as constraints. The most common approach is to calculate these ranges from the payoff table that has been demonstrated in Ref. [34].

It is noted that in the MMP problem of transmission expansion planning, only the ranges of objective functions: F_2, F_3 and F_4 are calculated, since F_1 is the main objective function. Then, these ranges for F_2, F_3 and F_4 are divided by q_2, q_3 and q_4 (equal intervals using (q_2-1), (q_3-1) and (q_4-1) intermediate equidistant grid points), respectively. Considering the minimum and maximum values of the ranges, we have the total of (q_2+1), (q_3+1) and (q_4+1) grid points for F_2, F_3 and F_4, respectively. Thus, we should solve $(q_2+1)\times(q_3+1)\times(q_4+1)$ optimization sub-problems, where sub-problem (i,j,l) has the following form:

$$\begin{aligned}
&\min\ F_1(x)\\
&\text{s.t.}:\ F_2(x)\leqslant\varepsilon_{2i}\ \ F_3(x)\leqslant\varepsilon_{3j}\ \ F_4(x)\leqslant\varepsilon_{4l} \qquad (10)\\
&\quad \varepsilon_{2i}=\text{Max}(F_2)-\left(\frac{\text{Max}(F_2)-\text{Min}(F_2)}{q_2}\right)\times i\ \ i=0,1,\ldots,q_2\\
&\quad \varepsilon_{3i}=\text{Max}(F_3)-\left(\frac{\text{Max}(F_3)-\text{Min}(F_3)}{q_3}\right)\times j\ \ j=0,1,\ldots,q_3\\
&\quad \varepsilon_{4i}=\text{Max}(F_4)-\left(\frac{\text{Max}(F_4)-\text{Min}(F_4)}{q_4}\right)\times l\ \ l=0,1,\ldots,q_4
\end{aligned}$$

where Max(.) and Min(.) represent the maximum and minimum values of the individual objective functions based on the payoff table, respectively. By solving each optimization subproblem, one Pareto optimal solution is obtained. Some of these $(q_2+1)\times(q_3+1)\times(q_4+1)$ optimization subproblems may have infeasible solution spaces, which will be discarded. A desirable characteristic of the ε-constraint MMP method is that we could control the density of the efficient set representation by properly assigning the values to the q_2, q_3, and q_4 [35]. The higher the number of grid points the more dense is the representation of the efficient set, but with the cost of higher computation times. In this paper, the number of intervals for the objective functions F_2, F_3, and F_4 is considered 4, i.e., $q_2=q_3=q_4=3$. After obtaining the Pareto optimal solutions by solving the optimization subproblems, the decision-maker needs to choose the best compromise solution according to the specific preference for different applications.

Using an AC power flow, the Security-Constrained TEP (SC-TEP) problem can be formulated as follows:

Subject to the following equality and inequality constraints:

$$Minimize\begin{cases} F_1 = IC = \underbrace{\sum_{t=1}^{T}(1-I)^{t\times TD}\sum_{l=1}^{nc} IC_l u_{lt}}_{Investment\ \mathrm{Cost(IC)}} \\ F_2 = EOC = \underbrace{\sum_{s=1}^{ns}\pi_s\left\{\underbrace{\sum_{t=1}^{T}(1-I)^{t\times TD}\left[TD\times 8760\times\sum_{g=1}^{ng}C_{2g}P^2_{G,gst}+C_{1g}P_{G,gst}+C_{0g}\right]}_{Operation\ \mathrm{Cost\ (OC)}}\right\}}_{Expected\ \mathrm{Operation\ Cost(EOC)}} \\ F_3 = ELSC = \underbrace{\sum_{s=1}^{ns}\pi_s\left\{\underbrace{\sum_{t=1}^{T}(1-I)^{t\times TD}\left[\sum_{i=1}^{nb}\alpha_i rp_{ist}+\beta_i rq_{ist}\right]}_{Load\ \mathrm{Shedding\ Cost(LSC)}}\right\}}_{Expected\ \mathrm{Load\ Shedding\ Cost(ELSC)}} \\ F_4 = -ELF = -\sum_{s=1}^{ns}\pi_s\sum_{t=1}^{T}LF_{st} \end{cases} \quad (11)$$

Equality constraints for current operating point and maximum loading point for both normal and contingency conditions are as given below:

$$P^c_{G,ist} - P_{D,ist} + rp^c_{ist} = \sum_{l\in EL}P^{0,c}_{L,lst} + \sum_{l\in CL}P^c_{L,lst} \quad \forall i\in B, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C, \mathrm{l}\in(i,j) \quad (12)$$

$$\widetilde{P}^c_{G,ist} - \widetilde{P}^c_{D,ist} = \sum_{l\in EL}\widetilde{P}^{0,c}_{L,lst} + \sum_{l\in CL}\widetilde{P}^c_{L,lst} \quad \forall i\in B, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C, \mathrm{l}\in(i,j) \quad (13)$$

$$Q^c_{G,ist} - Q_{D,ist} + rq^c_{ist} = \sum_{l\in EL}Q^{0,c}_{L,lst} + \sum_{l\in CL}Q^c_{L,lst} \quad \forall i\in B, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C, \mathrm{l}\in(i,j) \quad (14)$$

$$\widetilde{Q}^c_{G,ist} - \widetilde{Q}^c_{D,ist} = \sum_{l\in EL}\widetilde{Q}^{0,c}_{L,lst} + \sum_{l\in CL}\widetilde{Q}^c_{L,lst} \quad \forall i\in B, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C, \mathrm{l}\in(i,j) \quad (15)$$

$$P^{0,c}_{L,lst} = |V^c_{ist}|^2G^0_{ij} - |V^c_{ist}|\left|V^c_{jst}\right| \times \left(G^0_{ij}\cos\left(\theta^c_{ist}-\theta^c_{jst}\right) + B^0_{ij}\sin\left(\theta^c_{ist}-\theta^c_{jst}\right)\right) \quad \forall \mathrm{l}\in \mathrm{EL}, \ \forall s\in S, \ \forall t\in Y, \ \forall c\in C \ \forall i, \ j\in B, \mathrm{l}\in(i,j) \quad (16)$$

$$\widetilde{P}^{0,c}_{L,lst} = \left|\widetilde{V}^c_{ist}\right|^2G^0_{ij} - \left|\widetilde{V}^c_{ist}\right|\left|\widetilde{V}^c_{jst}\right| \times \left(G^0_{ij}\cos\left(\tilde{\theta}^c_{ist}-\tilde{\theta}^c_{jst}\right) + B^0_{ij}\sin\left(\tilde{\theta}^c_{ist}-\tilde{\theta}^c_{jst}\right)\right) \quad \forall \mathrm{l}\in \mathrm{EL}, \ \forall s\in S, \ \forall t\in Y, \ \forall c\in C \ \forall i,j\in B, \mathrm{l}\in(i,j) \quad (17)$$

$$P^c_{L,lst} = u_{lt}\left(\left|V^c_{L,lst}\right|^2G_{ij} - |V^c_{ist}|\left|V^c_{jst}\right| \times \left(G_{ij}\cos\left(\theta^c_{ist}-\theta^c_{jst}\right) + B_{ij}\sin\left(\theta^c_{ist}-\theta^c_{jst}\right)\right)\right) \quad \forall \mathrm{l}\in \mathrm{CL}, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad \forall i,j\in B, \mathrm{l}\in(i,j) \quad (18)$$

$$\widetilde{P}^c_{L,lst} = u_{lt}\left(\left|\widetilde{V}^c_{ist}\right|^2G_{ij} - \left|\widetilde{V}^c_{ist}\right|\left|\widetilde{V}^c_{jst}z\right| \times \left(G_{ij}\cos\left(\tilde{\theta}^c_{ist}-\tilde{\theta}^c_{jst}\right) + B_{ij}\sin\left(\tilde{\theta}^c_{ist}-\tilde{\theta}^c_{jst}\right)\right)\right) \quad \forall \mathrm{l}\in \mathrm{CL}, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad \forall i,j\in B, \mathrm{l}\in(i,j) \quad (19)$$

$$Q^{0,c}_{L,lst} = -|V^c_{ist}|^2B_{ij} - |V^c_{ist}|\left|V^c_{jst}\right| \times \left(G^0_{ij}\sin\left(\theta^c_{ist}-\theta^c_{jst}\right) - B^0_{ij}\cos\left(\theta^c_{ist}-\theta^c_{jst}\right)\right) \quad \forall \mathrm{l}\in \mathrm{CL}, \ \forall s\in S, \ \forall t\in Y, \ \forall c\in C \ \forall i,j\in B, \mathrm{l}\in(i,j) \quad (20)$$

$$\widetilde{Q}^{0,c}_{L,lst} = -\left|\widetilde{V}^c_{ist}\right|^2B_{ij} - \left|\widetilde{V}^c_{ist}\right|\left|\widetilde{V}^c_{jst}\right| \times \left(G^0_{ij}\sin\left(\tilde{\theta}^c_{ist}-\tilde{\theta}^c_{jst}\right) - B^0_{ij}\cos\left(\tilde{\theta}^c_{ist}-\tilde{\theta}^c_{jst}\right)\right) \quad \forall \mathrm{l}\in \mathrm{CL}, \ \forall s\in S, \ \forall t\in Y, \ \forall c\in C \ \forall i,j\in B, \mathrm{l}\in(i,j) \quad (21)$$

$$Q^c_{L,lst} = u_{lt}\left(-|V^c_{ist}|^2B_{ij} - |V^c_{ist}|\left|V^c_{jst}\right| \times \left(G_{ij}\sin\left(\theta^c_{ist}-\theta^c_{jst}\right) - B_{ij}\cos\left(\theta^c_{ist}-\theta^c_{jst}\right)\right)\right) \quad \forall \mathrm{l}\in \mathrm{CL}, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad \forall i,j\in B, \mathrm{l}\in(i,j) \quad (22)$$

$$\widetilde{Q}^c_{L,lst} = u_{lt}\left(-\left|\widetilde{V}^c_{ist}\right|^2B_{ij} - \left|\widetilde{V}^c_{ist}\right|\left|\widetilde{V}^c_{jst}\right| \times \left(G_{ij}\sin\left(\tilde{\theta}^c_{ist}-\tilde{\theta}^c_{jst}\right) - B_{ij}\cos\left(\tilde{\theta}^c_{ist}-\tilde{\theta}^c_{jst}\right)\right)\right) \quad \forall \mathrm{l}\in \mathrm{CL}, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad \forall i,j\in B, \mathrm{l}\in(i,j) \quad (23)$$

Inequality constraints for current operating point and maximum loading point for both normal and contingency conditions are as given below:

$$\sum_{t=1}^{T}u_{lt} \leqslant \mathrm{N}_l \quad \forall \mathrm{l}\in \mathrm{CL} \quad (24)$$

$$P^{\min}_{G,g} \leqslant P^c_{G,gst} \leqslant P^{\max}_{G,g} \quad \forall \mathrm{g}\in \mathrm{G}, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad (25)$$

$$P^{\min}_{G,g} \leqslant \widetilde{P}^c_{G,gst} \leqslant P^{\max}_{G,g} \quad \forall \mathrm{g}\in \mathrm{G}, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad (26)$$

$$Q^{\min}_{G,g} \leqslant Q^c_{G,gst} \leqslant Q^{\max}_{G,g} \quad \forall \mathrm{g}\in \mathrm{G}, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad (27)$$

$$Q^{\min}_{G,g} \leqslant \widetilde{Q}^c_{G,gst} \leqslant Q^{\max}_{G,g} \quad \forall \mathrm{g}\in \mathrm{G}, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad (28)$$

$$\left(P^{0,c}_{L,lst}\right)^2 + \left(Q^{0,c}_{L,lst}\right)^2 \leqslant \left(AP^{\max}_{L,l}\right)^2 \quad \forall \mathrm{l}\in EL, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad (29)$$

$$\left(\widetilde{P}^{0,c}_{L,lst}\right)^2 + \left(\widetilde{Q}^{0,c}_{L,lst}\right)^2 \leqslant \left(AP^{\max}_{L,l}\right)^2 \quad \forall \mathrm{l}\in EL, \quad \forall s\in S, \quad \forall t\in Y, \quad \forall c\in C \quad (30)$$

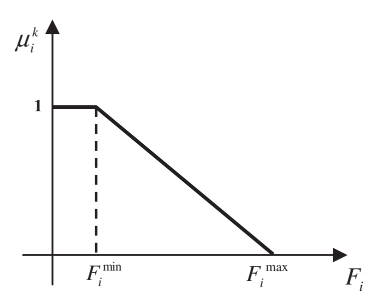

Fig. 2. Linear membership function of the *i*th objective.

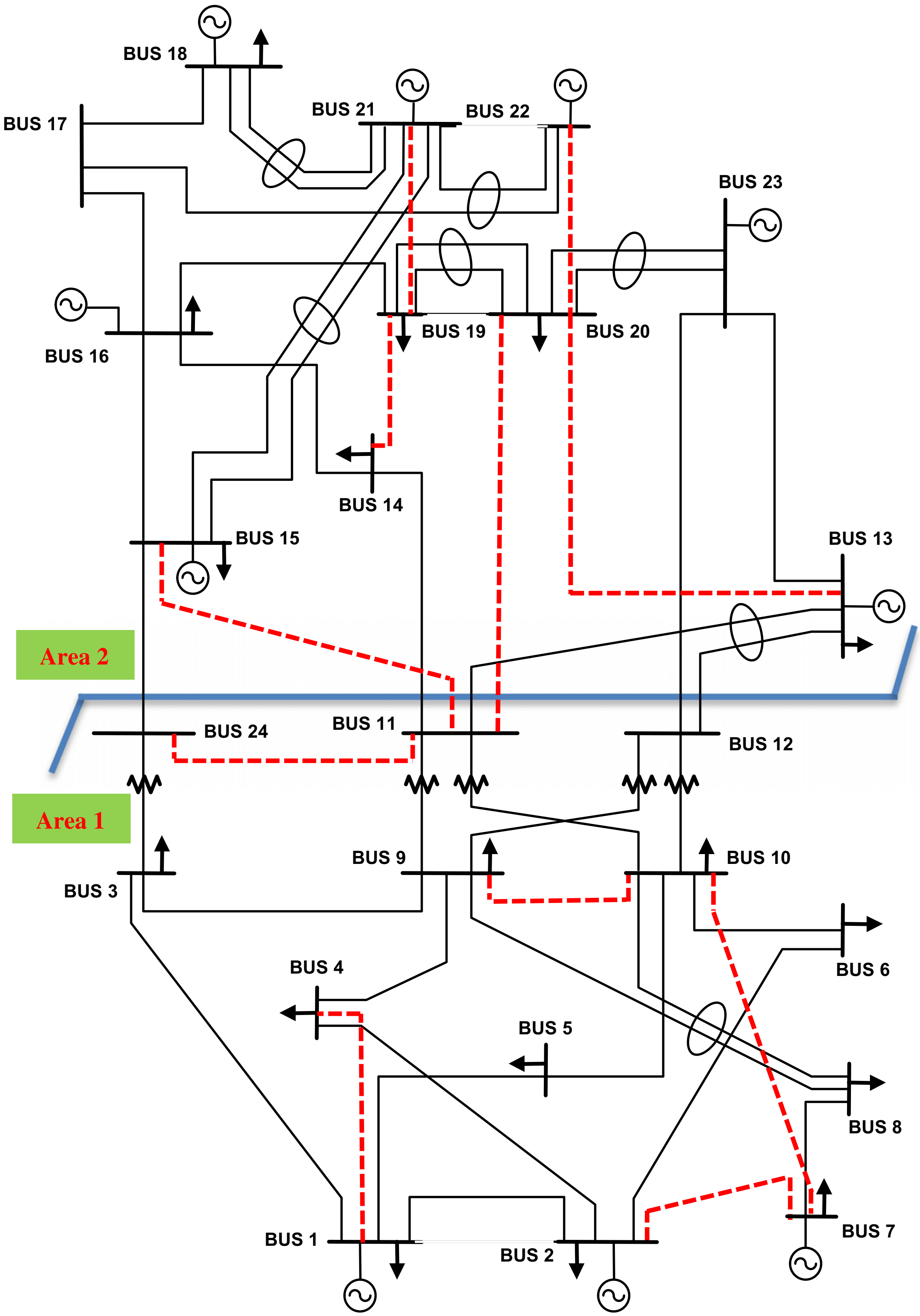

Fig. 3. Modified IEEE 24 bus reliability test system (MRTS).

$$\left(P^c_{L,lst}\right)^2 + \left(Q^c_{L,lst}\right)^2 \leqslant u_{lt}\left(AP^{\max}_{L,l}\right)^2 \quad \forall l \in CL, \quad \forall s \in S, \quad \forall t \in Y, \quad \forall c \in C \tag{31}$$

$$\left(\widetilde{P}^c_{L,lst}\right)^2 + \left(\widetilde{Q}^c_{L,lst}\right)^2 \leqslant u_{lt}\left(AP^{\max}_{L,l}\right)^2 \quad \forall l \in CL, \quad \forall s \in S, \quad \forall t \in Y, \quad \forall c \in C \tag{32}$$

$$\left|V^{\min}_i\right| \leqslant \left|V^c_{ist}\right| \leqslant \left|V^{\max}_i\right| \quad \forall i \in B, \quad \forall s \in S, \quad \forall t \in Y, \quad \forall c \in C \tag{33}$$

$$\left|V^{\min}_i\right| \leqslant \left|\widetilde{V}^c_{ist}\right| \leqslant \left|V^{\max}_i\right| \quad \forall i \in B, \quad \forall s \in S, \quad \forall t \in Y, \quad \forall c \in C \tag{34}$$

$$\widetilde{P}^c_{G,ist} = (1 + LF^c_{st} + K_G)P^c_{G,ist} \quad \forall i \in B, \quad \forall s \in S, \quad \forall t \in Y, \quad \forall c \in C \tag{35}$$

$$\widetilde{P}^c_{D,ist} = (1 + LF^c_{st})P_{D,ist} \quad \forall i \in B, \quad \forall s \in S, \quad \forall t \in Y, \quad \forall c \in C \tag{36}$$

$$\widetilde{Q}^c_{D,ist} = (1 + LF^c_{st})Q_{D,ist} \quad \forall i \in B, \quad \forall s \in S, \quad \forall t \in Y, \quad \forall c \in C \tag{37}$$

$$0 \leqslant rp^c_{ist} \leqslant rp^{\max}_i \tag{38}$$

$$0 \leqslant rq^c_{ist} \leqslant rq^{\max}_i \tag{39}$$

$$LF^{\min} \leqslant LF^c_{st} \leqslant LF^{\max} \tag{40}$$

It should be noted that in the above equations, $c = 0$ is related to the normal conditions of the power system. Eq. (11) shows the objective functions (i.e. Investment Cost (IC), Expected Operating Cost (EOC), Expected Load Shedding Cost (ELSC) and Expected Loading Factor (ELF), respectively). Eqs. (12) and (13) represent the active power balance at each bus. Eqs. (14) and (15) represent the reactive power balance at each bus. Constraints (16)–(23) indicate the active power and reactive power flows from existing and candidate lines, respectively. Superscripts indices 0 are used to denote the existing lines. The upper limits of the lines that could be added between two buses are represented by Eq. (24). Active and reactive power generation limits of the generators are represented by Eqs. (25)–(28). Transmission flow limits are shown by Eqs. (29)–(32) for the existing and candidate lines. The voltage constraints are shown by Eqs. (24) and (25). Constraints (26)–(28) show the loading factor (LF) equations. Eqs. (38) and (39) represent the upper limits on the active and reactive power load shedding at each bus, respectively. These limits can be determined based on political, economical and environmental issues. The lower and upper limits of the LF have been represented in Eq. (40). All of the above equations have been defined for the current operating point and the maximum loading point for both normal and contingency conditions. Also, it is important to highlight that in Eqs. (3)–(6) if candidate line l is selected for time period t, it will be considered as an existing line for higher time period i.e. $t+1$,$t+2$ and so on. Therefore, binary variable u_{lt} can be 1 for time period t and 0 for time period $t+1$. It means only one candidate line is constructed during time period t and $t+1$. If u_{lt} is 1 for the both of time period, it means two candidate line is constructed during time period t and $t+1$.

After determining the set of Pareto optimal solutions, the decision maker (here, ISO/TSO) should select the most flexible and best compromise solution depending on the importance of each objective function (security aspects or economic issues). In order to choose the best solution a fuzzy satisfying decision making approach has been employed in this paper [34]. A membership function (μ_i) is defined for each of the objective functions, i.e., F_1, F_2, F_3, and F_4, according to the following equation:

$$\mu^k_i = \begin{cases} 1 & F^k_i \leqslant Min(F_i) \\ \frac{Max(F_i) - F^k_i}{Max(F_i) - Min(F_i)} & Min(F_i) \leqslant F^k_i \leqslant Max(F_i) \\ 0 & F^k_i \geqslant Max(F_i) \end{cases} \tag{41}$$

where F^k_i and μ^k_i represent the value of the ith objective function in the kth Pareto optimal solution and its membership function, respectively. Fig. 2 shows the graph of the defined linear membership function. The whole membership function of the kth Pareto optimal solution (μ^k) is calculated based on its individual membership functions as follows:

$$\mu^k = \frac{\sum_{i=1}^{p} w_i \mu^k_i}{\sum_{k=1}^{M}\sum_{i=1}^{p} w_i \mu^k_i} \tag{42}$$

where w_i is the weight of the i^{th} objective function in the MMP problem and M is the number of Pareto optimal solutions. In our model, $p = 4$. The weight (w_i) could be selected by the ISO based on the importance of economical issues and different security aspects.

It should be noted that the obtained solution is a practical feasible local optimum solution that meets the ISO's requirements and security constraints rather than a global optimum solution due to the non-convexity nature of the formulated problem. However, it should be noted that when one tries to solve a non-convex Mixed-Integer Non-Linear Programming (MINLP) optimization problem, there will be no guarantee to obtain the global optimum solution and this issue remains correct for the most practical optimization models in complex systems (i.e. power systems that use AC-OPF mathematical models) [36,37].

Table 1
Candidate lines data.

Candidate lines		Capacity (MW)	Resistance (p.u.)	Reactance (p.u.)	Investment cost ($\$10^6$US)
From	To				
1	4	175	0.0028	0.0144	7.72
2	7	175	0.0040	0.0202	10.84
7	10	175	0.0038	0.0193	10.32
9	10	175	0.0030	0.0154	8.26
11	15	500	0.0042	0.0212	11.36
11	20	500	0.0045	0.0231	12.39
11	24	500	0.0021	0.0106	5.68
13	20	500	0.0021	0.0106	5.68
14	19	500	0.0032	0.0164	8.78
19	21	500	0.0026	0.0135	7.23
20	22	500	0.0026	0.0135	7.23

4. Case study

The proposed SC-TEP model has been successfully applied to the modified IEEE 24 bus reliability test system as shown in Fig. 3 [38]. This system consists of 32 generation units (10 plants) ranging from 12 to 400 MW, 17 loads and 38 lines. Area 1 is the 138 kV sub-network and Area 2 is the 230 kV sub-network and 5 tie lines have connected these two areas. It is assumed that all transmission lines are constructed by one TSO or ISO. We have considered an electricity market where every generator submits its supply offer in the form of a quadratic function. The market is cleared by the ISO. A 5% annual interest rate is utilized to calculate the present value of the investment in the planning horizon. The candidate lines are represented in Fig. 3 by red dashed lines. Table 1 shows the candidate lines data. The investment cost of the every new line is proportional to its reactance [39,40]. There is only one type of line and up to two new lines could be constructed per new branch.

This problem was solved on a PC running the 64-bit windows operating system with Core 2 Duo CPU clocking at 4.00 GHz and 8 GB of RAM memory. The software used was DICOPT under 64-bit version of GAMS [41]. A future horizon of 2 time periods has been considered in this paper, that each time period consists of 6 years. The Monte Carlo simulations were executed 100 times in time period one. In each execution 17 random numbers were produced (related to 17 loads). In the next time periods, the Monte Carlo simulations were run 100 times for each scenario obtained in the previous time period. Therefore, 10^4 scenarios were generated in the original tree and the probability of each scenario was 10^{-4}.

The GAMS/SCENRED [42] was used for reducing the number of scenarios. The SCENRED is a tool for reducing scenarios using the random data processes. The scenario reduction algorithms provided by the SCENRED determine a scenario subset and assign optimal probabilities to the preserved scenarios.

The number of scenarios was reduced by 80%, lowering down to six scenarios. Then, running the suggested program under this circumstance ("red percentage = 0.80"), Table 2 lists scenarios together with their corresponding probabilities. The problem has 6028848 single equations, 4014144 single variables and 22 discrete variables.

In addition, in Eq. (42), the weighting factors were selected as follows: $w_1 = 0.4$, $w_2 = 0.3$, $w_3 = 0.15$, $w_4 = 0.15$. In scenario generation by the MC, it should be noted that the load at bus i in time period t must be greater than the load at bus i in time period $t-1$, e.g. $P_{D,\,ist} \geqslant P_{D,\,is(t-1)}$. Also, it is important to note that the load growth of reactive power was considered similar to the load growth of active power. That is, the load power factor (pf_i) at each bus (in time period t and scenario s) was considered constant. In mathematical expression we have: $pf_i = \frac{P_{D,ist}}{\sqrt{P_{D,ist}^2 + Q_{D,ist}^2}} = \text{const.}\ \forall i \in B, \forall s \in S,\ \forall t \in Y$

The growth rate of the expected load was assumed 25% for all loads. That is, the expected amount of each load in period 2 is 25% more than its expected amount in period 1. Table 3 shows the PDF of the loads in time period 1 and the operation cost of each generating unit. With the aim of stressing the transmission lines, load and generation capacity during the first time period at each bus of the system was increased by a factor of 50%, i.e. the mean value of load in Table 3 is equal to the value of load in Ref. [38] that has been multiplied by 1.5.

Table 3
Generators and loads data.

Bus	Generator					Load
	Name		Quadratic offer function coefficients			P(MW)
			c_{2g}	c_{1g}	c_{0g}	
1	G_1	1	0	130	400.6849	$N \sim (162,12.9)$
		2	0	130	400.6849	
		3	0.014142	16.0811	212.3076	
		4	0.014142	16.0811	212.3076	
2	G_2	5	0	130	400.6849	$N \sim (145.5,12.5)$
		6	0	130	400.6849	
		7	0.014142	16.0811	212.3076	
		8	0.014142	16.0811	212.3076	
3	–	–	–	–	–	$N \sim (270,14.0)$
4	–	–	–	–	–	$N \sim (111,10.0)$
5	–	–	–	–	–	$N \sim (106.5,9.5)$
6	–	–	–	–	–	$N \sim (204,13.3)$
7	G_3	9	0.052672	43.6615	781.521	$N \sim (187.5,12.1)$
		10	0.052672	43.6615	781.521	
		11	0.052672	43.6615	781.521	
8	–	–	–	–	–	$N \sim (256.5,14.1)$
9	–	–	–	–	–	$N \sim (262.5,15.6)$
10	–	–	–	–	–	$N \sim (292.5,15.0)$
11	–	–	–	–	–	–
12	–	–	–	–	–	–
13	G_4	12	0.00717	48.5804	832.7575	$N \sim (397.5,15.0)$
		13	0.00717	48.5804	832.7575	
		14	0.00717	48.5804	832.7575	
14	–	–	–	–	–	$N \sim (291,13.4)$
15	G_5	15	0.328412	56.564	86.3852	$N \sim (475.5,16.7)$
		16	0.328412	56.564	86.3852	
		17	0.328412	56.564	86.3852	
		18	0.328412	56.564	86.3852	
		19	0.328412	56.564	86.3852	
		20	0.008342	12.3883	382.2391	
16	G_6	21	0.008342	12.3883	382.2391	$N \sim (150,11.8)$
17	–	–	–	–	–	–
18	G_7	22	0.000213	4.4231	395.3749	$N \sim (499.5,15.3)$
19	–	–	–	–	–	$N \sim (271.5,10.9)$
20	–	–	–	–	–	$N \sim (192,11.6)$
21	G_8	23	0.000213	4.4231	395.3749	–
22	G_9	24	0	0.001	0.001	–
		25	0	0.001	0.001	
		26	0	0.001	0.001	
		27	0	0.001	0.001	
		28	0	0.001	0.001	
		29	0	0.001	0.001	
23	G_{10}	30	0.008342	12.3883	382.2391	–
		31	0.008342	12.3883	382.2391	
		32	0.004895	11.8495	665.1094	
24	–	–	–	–	–	–

Table 4
Objectives optimal values for the single and multiple objective TEP.

Case	Objectives	
	Single objective	Multi-objective
IC ($)	2.84×10^7	1.05×10^8
EOC ($)	1.1260×10^9	1.0545×10^9
ELSC ($)	2.77×10^6	1.94×10^4
ELF (p.u.)	0.127	0.143

Table 4 shows the optimal results for single and multi-objective cases. The required solution time for the single objective case was about 6420s. In the single objective approach, investment costs of

Table 2
Preserved scenarios with corresponding probabilities.

Preserved scenarios (nodes)	n-392	n-709	n-1033	n-5604	n-7731	n-9238
Probability (π_s)	0.110	0.201	0.161	0.173	0.129	0.226

Table 5
Optimal objectives values for different sets of weighting factors.

	Case 1				Case 2				Case 3			
	$W_1 = 0.6$	$W_2 = 0.4/3$	$W_3 = 0.4/3$	$W_4 = 0.4/3$	$W_1 = 0.5$	$W_2 = 0.5/3$	$W_3 = 0.5/3$	$W_4 = 0.5/3$	$W_1 = 0.4$	$W_2 = 0.6/3$	$W_3 = 0.6/3$	$W_4 = 0.6/3$
IC ($)	8.64×10^7				8.64×10^7				9.78×10^7			
EOC ($)	1.2113×10^9				1.21657×10^9				1.1428×10^9			
ELSC ($)	1.53×10^4				2.10×10^4				1.03×10^4			
ELF (p.u.)	0.152				0.132				0.157			

Table 6
Optimal binary variables related to the candidate lines.

Lines	Case			
	Single objective		Multi-objective	
	Time period 1	Time period 2	Time period 1	Time period 2
1 (1–4)	0	0	0	0
2 (2–7)	0	0	0	0
3 (7–10)	0	0	0	0
4 (9–10)	0	0	0	0
5 (11–15)	0	0	0	1
6 (11–20)	1	0	0	0
7 (11–24)	0	0	0	0
8 (13–20)	0	0	1	0
9 (14–19)	0	1	0	1
10 (19–21)	0	0	1	0
11 (20–22)	0	1	0	1

candidate line are only considered as objective function and then EOC, ELSC and ELF are calculated based on the optimized variables obtained. In the multi-objective optimization case study 64 subproblems had to be solved by the GAMS/DICOPT solver. Among these optimization problems, 11 problems had infeasible solutions and so were discarded. Using the fuzzy trade-off method (described in the previous section), the best solution was obtained. The required solution time for the multi-objective case was about 74 h. It is important to highlight this fact that in the planning studies the time and computational burden is not the main issue due to the current computational power and also availability of the sufficient time for the planning.

As it can be seen in Table 4, the proposed multi-objective security-constrained AC-OPF based TEP model enhanced the security of the power system. The results of Table 4 for multi-objective case clearly show that the defined objectives in Eq. (11) are conflicting with each other. It means the more security can be obtained but with the more investment cost for constructing new transmission lines. The degree of the voltage security enhancement could be improved by increasing the related weighting factor. In the presented case studies the ELF was increased from 0.127 p.u. (in the single objective case) to 0.142 p.u. (in the multi-objective case) with $w4 = 0.15$. Also, the obtained results in Table 4 show that in spite of increasing the investment cost, the expected operation costs and the expected load shedding costs were reduced in the multi-objective case study compared with single objective case study.

To show the effects of various weighting factors on the optimal objectives values, the optimization problem was solved for different sets of weighting factor. Table 5 depicts the obtained results; it shows that if a higher voltage security margin (VSM) is required, then more investment costs should be provided. Table 5 demonstrates the conflicts among various objective functions. In Table 5 although the investment costs for case 1 and case 2 are similar (i.e. the same lines should be built for the both cases), EOC, ELSC and ELF are different. It means that there is always a trade-off among objective functions of EOC, ELSC and ELF. This trade-off can be handled by selecting the appropriate weighting factors depending on the importance of each objective by DM (decision maker).

Table 6 shows the optimal binary variables of the candidate lines in each time period for each method; number 1 means that the candidate transmission line should be constructed and number 0 means otherwise.

5. Conclusion

In this paper a new security-constrained TEP model was formulated and applied to the IEEE 24-bus RTS. The contributions of the proposed SC-TEP model were using AC-OPF instead of DC-OPF, considering four important objective functions including the VSM (to improve the expanded network voltage stability margins). N-1 contingency analysis was applied for testing and enforcing the adequacy of the expanded network and system loads was considered as random variables (with known PDFs). The random load scenarios were created using Monte Carlo simulations and then the low probability scenarios were omitted using a scenario reduction technique. The case studies results clearly showed that considering the VSM as an objective function in the TEP problem would improve the power system voltage stability margins by increasing the transmission investment costs. The developed SC-TEP model gives the network planner (e.g. ISO/TSO) the freedom to achieve the desired level of voltage security margin at its optimal transmission investment costs. The MODM (Multi-Objective Decision Making) was employed based on the ε-constraint method to solve the formulated multi-objective TEP problem and a fuzzy trade-off approach was used to choose the most preferred optimal solution from the Pareto fronts. The case studies results showed the applicability and efficiency of the proposed SC-TEP optimization model for the network planner.

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